# OK Geometry Basic <br> Part II <br> Reference for OK Geometry Sketch Editor (v. 17.2) <br> Zlatan Magajna <br> December 2019 

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## 1. Sketch Editor of OK Geometry

This document considers editing constructions in OK Geometry. Readers who are familiar with dynamic geometry programs Geogebra, Cabri, Cabri Express, Sketchometry, Cinderella, C.a.R. or JGEX may draw constructions in these programs and import them into OK Geometry. Making constructions in OK Geometry is, in general, similar to making them in other dynamic geometry software. OK Geometry allows many operations commonly found in dynamic geometry programs (e.g. dragging points, defining macros). Yet there are differences due to specific aims of OK Geometry. Dynamic geometry programs, above all, promote conceptual understanding. Thus, only very simple construction commands are available, for students should understand how to work out a other construction from basic operations. Furthermore, a lot of attention is put on specific visual representations that enable explorations and demonstrations. On the other hand, the aim of OK Geometry is to promote hypothetising, deductive reasoning and proving. Consequently, commands of OK Geometry Sketch editor allow quick and efficient design of geometric configurations. Besides direct non-trivial constructions (e.g. to construct a circle touching three given circles) Sketch editor allows also implicit constructions and constructions based on optimisation. Such constructions are obtained by imposing additional descriptive or optimisation requirements to an existing construction. For example, in a construction we want to achieve that a triangle is equilateral or that its area is minimal. Using specific tools (described in general help file of Ok Geometry) the student analyses the designed configuration, studies its properties, finds how to construct it with compass and ruler, and, possibly, also proves some property of the designed configuration.

Note that the Sketch editor is not available in Simple mode of OK Geometry.

### 1.1. The interface

The access to the Sketch mode of OK Geometry, i.e. the Sketch editor, is via the Sketch button in the main menu bar. When in Edit mode the Sketch button is sunken. To exit the Edit mode click the Sketch button again or some other button in the main menu bar.

The left pane of the interface (the information pane) contains various sections that are optionally shrunk or expanded (Figure 1). In the Task section you can write or edit the description of a task. The
Transformations \& Parameters section contains the lists of the transformations and parameters/expressions used or set in a construction. Finally, the Treatment section contains information to be used in the analysis (outside the Sketch editor). You can expand or shrink the sections by clicking on the triangle button on the title line of the section (right click on these buttons for a full expansion). The Help section contains the explanation of the used commands. Be aware of the expansions signs on the left side of the sections: click them to expand/shrink the sections, right-click them to fully expand the sections.

The right pane (visualisation pane) contains a drawing of the current construction. Above the drawing is the editor's menu line. Below the menu line is a very important (red coloured) information line. Keep an eye on it - it tells you what is going on and what you are expected to do. Additional help on how to use the commands is provided in the Help section (left pane) if it is expanded. On the right of the drawing pane there is a tool bar with some buttons for often used commands.


Figure 1

### 1.2. A first example

The following example will give you an idea how to make constructions in OK Geometry. We shall construct a (dynamic) right triangle with the inscribed circle in it. Furthermore, we shall label the points in which the circle touches the sides of the triangle. Here are the steps to follow.

1. Enter the Edit mode by pressing the Sketch button in the main menu bar. The editor's menu appears above the visualisation area. (If the current construction is not empty use the File /New project command in the main menu.)
2. To draw a right triangle select (click) the command Advanced/Shapes/Right triangle
3. The line below the menu (the information line) instructs you to pick three points (vertices): First a vertex at an acute angle, then the vertex at the right angle, and then the third vertex. The position of the third vertex will be automatically corrected so that the triangle will be right.
4. Label the vertices of the triangle. A simple way (but not the quickest) is to click the Autolabel button in the editor's toolbar (on the right of the information line, click the button with a letter on it). Then click the vertices in the desired order. The first vertex will be labelled $A$, the second $B$, etc. Alternatively, right-click somewhere on the visualisation pane. Select the Labels subcommand and then choose an appropriate labelling method, for example Label all.
5. To draw the inscribed circle, select in the editor's menu the command Circle/Circle 3obj. This command draws a circle that touches or passes through three given objects (lines, points, circles).
6. Click on each of triangle's three sides. After you pick the first side the whole triangle is emphasised since the triangle was constructed as one object). A circle appears.
7. There are several circles that touch the three baselines of the sides (from inside or outside the triangle). To access them click repeatedly the Alt button in the editor's toolbar. Click Alt several times (or use the mouse wheel) until you obtain the desired solution.
8. Finally, to draw the points where the circle touches the sides, select the command Point/Point or click on the Point button on the editor's toolbar. Click on the three points of touch. These points are automatically labelled.

The construction is complete and ready to be analysed (Figure 2) - just click on the main menu button Observe. Among other properties the analysis will spot the fact that the lines AD, BE, and CF meet at the same point (a well know fact to anybody who has heard of Gergonne point).


Figure 2

## 2. General guidelines

Most commands of the OK Geometry Sketch editor are rather straightforward and there is no reason to study them in detail at first reading. However, we strongly suggest to read the general guidelines below for specific properties of OK Geometry Sketch editor.

With the editor you can edit points, lines, line segments, polylines (polygonal lines), circles, arcs, conics, transformations, and numeric parameters. Polylines are chains of line segments. Segments are thus a special case of polylines. Note that OK Geometry does not operate with rays and vectors.

### 2.1. Finding commands

In order to allow fast and efficient constructing, OK Geometry Sketch editor contains many commands. The commands can be accessed via the toolbox on the right part of the editing pane (note that more buttons can be accessed with a right click on buttons), see Section 3, or via commands in the menu line. Though the commands and the positions of the commands in the menu follow some logical order, a beginner may have problems in finding them and using them. To overcome this you can use the Search command help in the Help menu. We explain this help with an example. Suppose you want to obtain a line that is tangent to two circles. In the Search command prompt write an approximate name or word related to the command, for example tangent. In the form that appears you find a list of commands related to your prompt. A click on each of the commands gives you (Figure 3)

- the position of the command in the menu,
- the parameters,
- important notes regarding the use of Alt and Anchor modifiers (see 2.5 and 2.1),
- a description of the command,
- a pictorial explanation (click on the >>> sign for more illustrations).

In our case, we see that the sought command is Line: 2 objects. A right click on the Description part activates the command (when Sketch mode).


Figure 3

### 2.2. The blue dot

The blue dot in the upper left part of the construction pane serves for two purposes: 1. it indicates the perturbation status of the displayed construction; 2. allows a quick configuration of the displayed construction.

1. it indicates the mode of displaying the construction. OK Geometry displays constructions in two modes. In the exact mode the positions of points is as is in the construction (e.g. as imported from some system). In this case there is one red point in the centre of the blue dot. In the perturbed mode the represented construction is one of the perturbed variants of the original construction. In this case there are several random red points in the blue dot. The size of the blue dot shows the size of the perturbation area around free points. A click on the blue dot changes the modes from one to the other.

Internally, all observations are made in the perturbed mode; On the other side, all exports of constructions are made in the exact mode, irrespective of the status of the blue dot.

Note that some constructions cannot be displayed in exact mode.

In general, it is rather irrelevant which mode is active. In the exact mode you see horizontal and vertical lines as horizontal and vertical. Furthermore, in the exact mode there is a mechanism that allows an easy construction of horizontal and vertical lines (though they need not be horizontal or vertical by construction). On the other hand, in the perturbed mode the position of all free points often changes slightly.
2. A right click on the blue dot open a form the configuration of some often used parameters related to the display of the configuration (e.g. the way of filling areas, font of point labels). In particular, you can also set the level of the displayed help for the construction commands.

### 2.3. Command first, then parameters

Editor's action are started by activation the appropriate command (the involved objects are chosen later). The information line tells you what you are expected to do when a command is active. The active command (together with required actions) is displayed in the information line just below the editor menu. It is a good practice to constantly check the information line, especially when you learn to use the program. Most commands remain active until either another command is activated or Esc is pressed. For example: To draw some points first click the command Point/Point and then click on the desired positions of one or more points. Similarly, to drag a point it is necessary to first click the command Action/Drag Point and then drag one or more points.
The execution of any command can be cancelled any time by pressing Escape key (Esc).

### 2.4. Only labelled points count

The label of a point consists of a single letter eventually followed by a numeral or apostrophes. You can thus use labels like: A, A2, A', k3, u1'". Label consisting of more than one character are also admitted, OK Geometry will, where necessary, write the label in parenthesis, (AE) for AE. Usually you will label only
the points that are potentially important for the analysis. Labels are case sensitive. The characters $x, y, z$ are not admitted in labels.

Note. You can label also other objects (lines, circles, etc.) - either by writing the name immediately after the object is generated or you can do it later as textual information (see Mark|Text command).
However, only labels of points are considered in the analysis of the construction, all other labels are ignored
Note. If a command results in a construction of a single point, this point is usually automatically labelled. It is possible to write the label of an object immediately after it is constructed. The position of the label can be modified with mouse wheel or Anchor command. Obviously, the label and the position of the label can be modified later with appropriate commands.

### 2.5. Alternative solutions

Use Alt button on the editor's toolbar ${ }^{1}$ (possibly together with mouse wheel) for alternative results of operations. Some constructions admit two or more solutions in this case a red coloured Alt button appears on the editor's toolbar. Only one solution is shown at time. By repeatedly pressing Alt button alternative solutions can be accessed. Note that some alternatives may be 'empty' or duplicated. For example, the task of constructing a circle that is tangent to three circles has usually several solutions. Only one is drawn at time. By repeatedly pressing Alt button one obtains the desired solution. For convenience Alt button executes also analogies of some commands (e.g. parallel/perpendicular line). Figure 4 shows a simple example (line tangent to two circles) with several possible solutions including the radical axis. Using Alt it is possible to select the desired solution.


Figure 4

### 2.6. Safe points

Safe points is a feature that determines the behaviour of a point that is positioned on an object, e.g. line segment, segment of polyline, circular arc. Safe points feature can be turned ON or OFF. If Safe points is ON then line segments (segments of a polyline, circular arcs) in a construction of a point are treated as respective base objects. For example, a free point on a line segment is treated as a point on the baseline of the line segment, thus it can be dragged along the line outside the line-segment. The intersection point of two line segments, constructed as intersection under Safe points ON, is constructed as the

[^0]intersection of the respective baselines. We illustrate this feature with two examples (Figure 5).

| Construction of points | Behaviour of the construction |
| :---: | :---: |
|  <br> G was defined as intersection of segments with Safe points OFF. <br> H was defined as intersection of segments with Safe points ON. | When $A$ is dragged so that the segment $A B$ does not intersect the polyline CDE, the point G 'disappears'. Since H was constructed under Safe points $\mathrm{ON}, \mathrm{H}$ is positioned at the intersection of the respective base-lines of the segments $A B$ and CD. (Note. The dotted lines are displayed here only for evidence.) |
| G was defined as a point on the polyline ABCA with Safe points OFF. <br> H was defined as a point on the polyline ABCA with Safe points N. | As $G$ is dragged around, it remains on the polyline, since it was constructed under Safe point OFF. The point H was constructed under Safe points ON, thus it was positioned on the line AB. When dragged H remains on the line AB. (Note. The dotted line is displayed here only for evidence.) |

Figure 5

Whether a point is constructed with Safe points ON or OFF it depends on the type of intersection or linking one needs. The ON option is more robust in the sense that the intersection of two line segments always exists (unless the segments are parallel).

### 2.1. The anchor command

The Anchor command and button $\pm$ modifies a relevant element of various object. The table explains and illustrates, which element are modified in various types of objects. The command is used repeatedly until the desired effect is achieved - a quicker way is to use the scroll button on the mouse.

| Object | Effect | Example before | Example after (possibly <br> repeated) anchor |
| :--- | :--- | :--- | :--- |
| Point (label) | Changes the <br> position of the <br> label wrt. the <br> point. |  |  |
| Angle | Sets a different <br> angle with the <br> same base-lines <br> as the original. |  |  |
| Arrowed |  |  |  |
| segment | Modifies the <br> position of the <br> arrows. |  |  |
| Text/marker | Changes the <br> position of the <br> text/marker wrt. <br> the reference <br> point. |  |  |

### 2.2. Geometric transformations and numeric parameters

Geometric transformations and numeric parameters are considered as explicit objects. For example, you may define a rotation around a point to be an object by itself. At any time you can apply this transformation to any object.

The used numeric parameters and transformations are displayed on the left hand side of the display. The numeric parameters may be assigned a desired value or may be the result of a measurement Numeric parameters can be 'dragged'. In expressions you can to parameter named 'mypar' as [mypar].

### 2.3. Commands on polylines

Commands that apply to line segments usually have, when applied to a polyline, two or three variants. Such commands can be applied to one segment of the polyline, to each segment of the polyline, or to the polyline as a whole. For example: The command Midpoint when applied to a polyline has three version:

- you can generate the midpoint of the segment you pointed to,
- you can generate the set of midpoints of each segment of a polyline,
- you can generate the midpoint of a polyline as a whole (i.e. at the point that splits the polyline into two parts of equal length).


### 2.4. Emphasized, bleached, and transparent objects

Important objects, e.g. those intended to represent some geometric property, can be declared as emphasized. In Sketch editor it is possible to control the emphasizing level with the Shift+Up/Down keys, with Shift+mouse wheel, or with the Scenes command; elsewhere the emphasizing level is controlled with sliders. The emphasizing level in icons can be changed at any time with sliders.

Objects that are relevant but may cause unwanted confusion can be declared as bleached. In this way they become less visible. In Sketch editor the degree of visibility (bleaching level) can be adjusted with Alt+Up/Down keys, with Alt + mouse wheel, or with the Scenes command.

Objects can be also declared as transparent. Such objects behave similarly as hidden objects. However, transparent objects are , contrary to hidden objects, considered in the observational analysis.
Transparent objects are mostly used in scenes. A complete construction is presented as several scenes, scenes differ among them only in the appearance of the objects: an object can be transparent in one scene and non-transparent in another, or can be emphasized in one scene and non-emphasized in other etc.

There are several commands for setting the relation-emphasis and bleaching status of objects. The usual way is to use the buttons for toggling the emphasis/bleaching status of objects.

Figure 6 displays a triangle ABC with bleached incircle and three relation-emphasized line segments. The three variations show different bleaching and emphasizing effects.


Figure 6

### 2.5. Scenes

A construction may consist of several scenes, which share a common underlying construction. The scenes of a construction vary only in selection of objects that are emphasised, transparent, and bleached. The scenes are visible (can be displayed) only in Project mode with big icons. The aim of the scenes is to catch the attention of different specific configurations in the construction.

The scenes of a construction are managed with the Scenes command and the self explaining forms that is displayed when the command is active.

## 3. Toolboxes

### 3.1. Visibility and current style information

On the upper right of the editor window there is a small toolbox setting the style (colour, widths, shape of lines/points, etc.) of the drawn objects. The toolbox contains also some visualisation related commands and indicators of mode of work.

| Button | Command | See cmd |
| :---: | :---: | :---: |
| Cycl | Cyclic mode ON/OFF switch and indicator (Only in Plus mode). When the cyclic mode is ON , the program performs cyclic constructions on triangles. (See OK Geometry Plus documentation for details.) |  |
| Safe | Safe point button ON/OFF switch and indicator. See Section2.6. |  |
| H B T N X M | Visibility ON/OFF switches and indicators. <br> When the switch $\mathbf{H}$ (hidden) is $\mathbf{O N}$, all hidden objects are displayed as if they were not hidden. <br> When the switch B (bleach) is ON, bleached objects are displayed as if they were not bleached, i.e. the objects are well visible. <br> When the switch $\mathbf{T}$ (text) is ON , the texts on the construction are displayed. Note that labels of points of objects are always visible, regardless this switch. <br> When the switch $\mathbf{X}$ is $\mathbf{O N}$, the auxiliary lines of angles are displayed. <br> When the switch $\mathbf{M}$ is $\mathbf{O N}$, all markers in the construction are displayed. |  |
| $\bigcirc$ | The rightmost indicator displays the style of new objects to be created in Sketch editor. To change the style of new objects, click on the indicator and set the styles in the form that appears. |  |

### 3.2. Sketch editor toolbox

The editor's tool bar contains the more frequently used commands. When two buttons are shown together, a right click on the button switches between them.



|  | Changes the position of the label of a labelled point. Changes the selection of angle between intersecting lines. Changes the direction of arrow in an arrowed segment. Changes the position of a text wrt. its reference point. Changes fill state of a polygon (defined by a polyline). |  |
| :---: | :---: | :---: |
| $\cdots$ | Drag. Dynamically changes the construction as a selected point is dragged or a selected parameter is changed. The command is reset after each dragging. | 10 |
| [] | Zoom fit. Press repeatedly for various zooming options: view all objects; view all displayed points; view all labelled points; user (saved) view. |  |
| $\pm$ | Pan. Move the whole configuration. |  |
| 4 | Undo. Undoes the last command. Can be pressed several times in succession. | 46 |
| - | Redo. Redoes the last undone command. Can be pressed several times in succession. | 47 |
| $\otimes$ | Redefine object. Pick the object to be redefined and the set the new construction of the object. | 14 |
| 4 | Set cyclic objects. (Only in Plus mode) Declare 3 pointed objects as cyclic with respect to the reference triangle. |  |

Some examples of geometric constructions are described in detail in Section 5. All commands are explained in detail in Chapter3.

## 4. Description of the menu commands

### 4.1. Sketch editor: organisation of the menu

| 1. Action commands | The group contains commands for manipulating geometric objects: dragging points or parameters, labelling vertices, hiding and deleting objects, undoing commands, and zooming. |
| :---: | :---: |
| 2. Point commands | Here are various commands for positioning and constructing points and sets of points. |
| 3. Line commands | Here are various commands for constructing lines, line segments, and polylines. Note that OK Geometry does not explicitly use rays and vectors. |
| 4. Circle commands | Here are various commands for constructing circles, circular arcs, and conics. |
| 5. Number commands | This group contains commands for measuring geometric quantities, defining numeric values (as parameters), for calculating expressions, and for composing logical conditions. |
| 6. Transformation commands | Here are the commands for defining transformations and commands for applying transformations to objects. A transformation is treated as an explicit object. Once it is defined it can be applied later to various objects. |
| 7. Advanced commands | Here you can find commands for complex operations: for constructing commonly used shapes, for constructing a locus, for checking conditions, for fulfilling conditions (implicit constructions), for constructions based on optimisation. |
| 8. (Special commands) | This group of commands is available only in Plus mode. <br> Here you can find a wide range of commands and constructions related to triangle geometry. Useful only for specialist in triangle geometry. |

### 4.2. Action commands

| Command | Meaning | Comments |
| :--- | :--- | :--- |
| 1. New project | Clear the current <br> project and the <br> current <br> construction. |  |
| 9. New construction | Clear the current <br> construction. | Note. The project (ie. all icons) remains <br> unchanged. <br> Drag free points <br> Dynamically <br> change the value <br> of a parameter |
| Only free (non-constructed) points can be <br> dragged. <br> Note. A long (continuous) click on a free point <br> automatically call the dragging commant. <br> It is possible to 'drag' numeric parameters that <br> were explicitly set as constant numbers (and not <br> as expressions or as measurements). To change <br> the value of a parameter, activate the drag <br> command and then click a numeric parameter. <br> In the form that appears set the minimal, <br> maximal value of the parameter as well as the <br> incremental step and click Apply interval. <br> Change the value of the parameter with the <br> slider. |  |  |
| 11. Hide Show hidden |  | Unhide and hide <br> objects |
| All objects (displayed and hidden) are shown. <br> The hidden ones are of red colour. By clicking <br> the objects you change their status from hidden <br> to displayed and vice versa. |  |  |
| objects are taken into account in the tan |  |  |
| construction, but they are not visible and they |  |  |
| are not observed. To unhide objects use the |  |  |
| command Action/Show hidden hidden. |  |  |
| Note that parameters and transformations |  |  |
| cannot be hidden. |  |  |


| 13. Delete | Delete an object together with all objects that depend on it |  |
| :---: | :---: | :---: |
| 14. Redefine object | Redefine an object and modify accordingly the depending objects. | Pick the object to be redefined. Then define a new object in place of the redefined one. <br> Example. Suppose you draw a line AB. After several construction steps involving the line $A B$ you find out that AC should be AC used instead of AB. To correct the situation redefine the line: <br> - Select the command Redefine object <br> - Click the line AB. The line turns blue and dotted. <br> - Select the command Line 2pts <br> - Click the points A and C. The situation is corrected. |
| 15. Visually align 2 points | Visually align two points horizontally or vertically in exact (nonperturbated) view. | The command aligns two selected points horizontally or vertically, whichever is closer. The perfect alignment occurs only in exact (nonperturbated) view and in exported/printed images. <br> The points to be aligned should be 'free' and remain free - thus they are randomly perturbed during the observation and the alignment does not occur. However, after switching to exact view, the points are again aligned. |
| 16. Labels/Auto label | Auto label selected points Unlabel points | The point you click on is labelled with the first available label (as shown in the editor's toolbar). If the selected point is already labelled then the label is deleted. <br> To change the starting label press Alt-S or use the command Actions/Labels/Label vertex. |
| 17. Labels/Label vertex | Label selected points with specified labels | Click the point you want to label. Then write the desired label of the point. The label should comply with the format of labels (see General guidelines). |


| 18. Labels/Clear label | Delete the label <br> of selected <br> points. | Click the point to be unlabelled. |
| :--- | :--- | :--- |
| 19. Labels/ Anchor <br> label/angle/text | Modify the <br> position of labels <br> and text wrt. the <br> defining point <br> Select another <br> angle with same <br> baselines as a <br> given one <br> Change <br> congruence mark <br> Change arrows <br> on a segment <br> Change the fill <br> state of a region | Anchor can be pressed several times in <br> succession. Instead of repeated anchor the <br> mouse wheel can be used. <br> Anchor changes the position of the label of a <br> labelled point. Note that labels can be also <br> dragged' by a continuous click. <br> Anchor changes the selection of angle between <br> intersecting lines (there are 8 choices). <br> Anchor changes the size and direction of arrow <br> in an arrowed segment. <br> Anchor changes the position of a text wrt. its <br> reference point. <br> Anchor changes the fill state of a region(defined <br> by a polyline or circle. |
| 20. Labels/Set next label | Set new label for <br> auto-labelling | See Labels/Auto label. |
| 21. Labels/Label all | Auto label all <br> shown points | All displayed unlabelled points are labelled in <br> the same order as they were defined. Each <br> displayed point is named by the first available <br> label. |
| 22. Labels/Clear all labels | Clear all labels | The labels of all points are deleted. |

$\left.\left.\begin{array}{|l|l|l|}\hline \text { 23. Mark/Mark angle } & \text { Mark an angle } & \begin{array}{l}\text { To mark an angle select: 1. a point on the ray; 2. } \\ \text { the vertex of the angle; 3. a point on the other } \\ \text { ray. } \\ \text { Alternatively, select: 1. one of the baselines; 2. } \\ \text { another baseline of the angle. } \\ \text { A positively oriented angle is marked. } \\ \text { Note. Use Anchor command to mark other } \\ \text { angles related to the marked angle. } \\ \text { Note. The radius and the type of the arc } \\ \text { denoting the angle are set automatically. To } \\ \text { modify the radius and the type of angle mark, } \\ \text { right click the blue dot or use the command } \\ \text { Configuration/General options/Sketch. } \\ \text { Note. The angle marks can be limited to angles } \\ \text { less than 90, 180 or 360 degrees. To set this } \\ \text { right click the blue dot or use the command } \\ \text { Configuration/General options/Sketch. }\end{array} \\ \hline \text { 24. Mark/Mark area } & & \begin{array}{l}\text { Mark or unmark } \\ \text { the interior of a } \\ \text { polygon }\end{array} \\ \hline \begin{array}{l}\text { The command changes the fill state of a polygon } \\ \text { or circle. The colour of the filled interior is } \\ \text { determined by the colour of the edging polyline. }\end{array} \\ \hline \text { 25. Mark/Mark arrow } & \begin{array}{l}\text { Note. There are several ways of hatching (filling) } \\ \text { the area. To select the type and density of } \\ \text { hatching right click the blue dot or use the } \\ \text { command Configuration/General } \\ \text { options/Sketch. }\end{array} \\ \text { Mark one or two } \\ \text { arrows on a line } \\ \text { segment }\end{array} \quad \begin{array}{l}\text { Apply repeatedly for various arrow modes. } \\ \text { Adding arrows affects a line segment only } \\ \text { visually, not functionally (i.e. arrowed segment need not to be closed in order to } \\ \text { does not represent a vector). } \\ \text { be (area) filled. } \\ \text { Note. You may also use the Anchor command to } \\ \text { mark/unmark the area of a polygon. }\end{array} \right\rvert\, \begin{array}{l}\text { Note. You may also use the Anchor command to } \\ \text { vary the type of arrowing of a segment. }\end{array}\right\}$

| 26. Mark/Mark congruence | Mark various congruence marks on segments and angle marks | The command draws a congruence mark on a line segment or angle mark. Apply repeatedly for various congruence marks. <br> Note. You may also use the Anchor command to vary the type of congruence mark. |
| :---: | :---: | :---: |
| 27. Mark/Marker | Draw a marker with a ballooned comment | Markers are encircled numbers 1-10 with optional comments or predefined symbols like !, ?, *. <br> Note. You may also use the Anchor command to vary the position of the marker wrt. its reference point. |
| 28. Mark/Detour point | Make an object to visually avoid a given point | If a point lies on or near a line or circle it is possible to detour later so that it viaually avoids the point. This may be useful when presenting hypothetical situations ("Assume that the line does not pass through A ...") or facts to be yet proved ("We do not know yet if the line passes through A, we need to prove this."). <br> In order to detour an object away from a point: <br> 1. pick the object; 2. pick the point to be avoided. <br> Note. You may use the Alt command to vary the side of the detour. <br> Note. The radius of the detour can be set in Configuration/General options/Sketch. |
| 29. Mark/Detour segment | Make an object <br> visually avoid a <br> given point along <br> a segment | If a point lies on or near a line or circle it is possible to detour later so that it viaually avoids the point. This may be useful when presenting hypothetical situations ("Assume that the line does not pass through A ...") or facts to be yet proved ("We do not know yet if the line passes through A, we need to prove this."). <br> This command serves the same function as Detour point, only the presentation is a bit different. <br> In order to detour an object along a segment: 1. pick the object; 2. pick the beginning of the detoured segment; 3. pick the end of the detour. The picked points become hidden. Note. You may use the Alt command to vary the side of the detour. |


| 30. Mark/Text | Include a simple text in the construction | The command is used to insert simple text in the dynamic construction or to modify a previously inserted text. The text may spread over several lines. <br> Note. The non standard characters are displayed on the construction display. However, it is not granted they are correctly exported/printed. <br> Note. To insert a dynamic value of a parameter, put the parameter name in brackets. For example: if Length_DA is the name of a parameter, you may insert a text like "Length of DA is [Length_DA]". <br> Note. You may also use the Anchor command to vary the position of the text wrt. its reference point. |
| :---: | :---: | :---: |
| 31. Background | Set the background of the construction | The background is not intended to be the background of a construction to be observed since the constructed points are usually not fixed. <br> The background is rather intended to be a selfstanding imported image to be displayed. Eventually, add objects in exact (nonperturbated) mode. |
| 32. Background/Background from file | Import background from an image file | The background is not intended to be the background of a construction to be observed since the constructed points are usually not fixed. <br> The background is rather intended to be a selfstanding imported image to be displayed. Eventually, add objects in exact (nonperturbated) mode. |
| 33. Background/Background from clipboard | Import background from the image in the clipboard | The background is not intended to be the background of a construction to be observed since the constructed points are usually not fixed. <br> The background is rather intended to be a selfstanding imported image to be displayed. Eventually, add objects in exact (nonperturbated) mode. |
| 34. Background/Clear background | Clear the background |  |


| 35. Scenes... | Manage the scenes of the current construction | With this command we create various scenes related to the same construction. The basic scene, the one we work on when we do not edit scenes with the Scenes command, is called the ground scene. The scenes (up to 10 of them) share the same construction, however the objects in the construction are shown with different ways. A given object can be emphasised, bleached or be transparent (invisible). <br> With the self-explaining form that appears you can set emphasise/bleach/transparency attributes of the objects for scenes of the current construction as scenes. See Section Note. Scenes can be displayed in Project mode with big icons. You can review the scenes also with Commands\|Construction steps command in the main menu. <br> Note. When the Scene form is not present, only the ground scene is displayed. |
| :---: | :---: | :---: |
| 36. Emphasise/Toggle bleach | Toggle the bleach status of objects | A bleached object is displayed as less visible (fainted). <br> Note. You can vary the level of bleaching with Alt+Mouse-wheel or with an occasionally shown slider. <br> Note. The command affects only the ground scene. Use Scenes... command to modify other scenes of the construction. |
| 37. Emphasise\|All bleach | Bleach all objects | A bleached object is displayed as less visible (fainted). <br> Note. You can vary the level of bleaching with Alt+Mouse-wheel or with an occasionally shown slider. <br> Note. The command affects only the ground scene. Use Scenes... command to modify other scenes of the construction. |
| 38. Emphasise/Unmark all bleach | Un-bleach all objects | A bleached object is displayed as less visible (fainted). <br> Note. The command affects only the ground scene. Use Scenes... command to modify other scenes of the construction. |
| 39. Emphasise/Toggle emphasis | Toggle the emphasis status of object | An emphasised object is displayed in a way to be well visible in a controlled way. <br> Note. You can vary the level of bleaching with Shift+Mouse-wheel or with an occasionally shown slider. <br> Note. The command affects only the ground scene. Use Scenes... command to modify other scenes of the construction. |


| 40. Emphasise/Unmark all <br> emphasis | Display all objects <br> as non- <br> emphasised | An emphasised object is displayed in a way to be <br> well visible in a controlled way. <br> Note. The command affects only the ground <br> scene. Use Scenes... command to modify other <br> scenes of the construction. |
| :--- | :--- | :--- |
| 41. Emphasise/Toggle <br> transparent | Toggle the <br> transparency <br> status of object | A transparent object is considered in <br> observation but is not visible. This feature is <br> used when a construction is presented in several <br> scenes. <br> By default, transparent objects are displayed in <br> yellow and are not shown in icons. <br> Note. The command affects only the ground <br> scene. Use Scenes... command to modify other <br> scenes of the construction. |
| 42. Emphasise/Unmark all | Display all objects <br> as non- <br> transparent | A transparent object is considered in <br> observation but is not visible. This feature is <br> used when a construction is presented in several <br> scenes. <br> By default, transparent objects are displayed in <br> yellow and are not shown in icons. <br> Note. The command affects only the ground <br> scene. Use Scenes... command to modify other <br> scenes of the construction. |
| 43. Emphasise/Make |  |  |
| everything transparent | Display all objects <br> as non- <br> transparent | A transparent object is considered in <br> observation but is not visible. This feature is <br> used when a construction is presented in several <br> scenes. <br> By default, transparent objects are displayed in <br> yellow and are not shown in icons. <br> Note. The command affects only the ground <br> scene. Use Scenes... command to modify other <br> scenes of the construction. |
| 44. Emphasise/Use current |  |  |
| attributes of object in all |  |  |
| scenes |  | In the various scenes of a construction each <br> object has in each scene has a certain state of <br> transparency, emphasis or bleach. With this <br> ommand we copy these statuse of a picked <br> object in the ground scene to all scenes. <br> For example, if a pointed line is not transparent, <br> bleached and not emphasised in the ground <br> scene, then the same holds for the line in all <br> scenes of the construction. |


| 45. Restyle objects | Set the visual appearance of picked objects. | In the form that appears select the width, colour, and shape of line/point. Note that you may modify only some attributes and leave others unchanged. <br> Then click on the objects you want to apply the selected appearance. <br> The More command allows efficient ways of simultaneously restyle several objects. It is possible to restyle at once all objects of a given colour, or all objects of a given type and pattern, etc. |
| :---: | :---: | :---: |
| 46. Undo command | Undo the last command | The command can be repeated several times. |
| 47. Redo command | Undo the last undo command | The command can be repeated several times. |
| 48. Zoom/Zoom in [Ctrl+Plus] | Zoom in with respect to the centre of the displayed area | Note. To achieve a similar effect use Ctrl+Mouse-wheel. |
| 49. Zoom/Zoom out [Ctrl+Minus] | Zoom out with respect to the centre of the displayed area | Note. To achieve a similar effect use Ctrl+Mouse-wheel. |
| 50. Zoom/Pan view | Drag the view area to the desired position | Note. To achieve a similar effect use a long (continuous) click on an empty space in the construction. |
| 51. Zoom/Zoom view | Mark the region to be zoomed-in | Define (by dragging) the rectangle, which becomes the new view area. |
| 52. Zoom/Zoom save [Ctrl+End] | Save the current view | The saved view can be accessed by repeatedly pressing the [] button in the Sketch editor tool box. |


| 53. Zoom/Fit points/objects <br> $[$ Ctrl+Home] | Zoom to various <br> ways of showing <br> all points/objects | Sets the viewed area so that all visible points are <br> displayed. If repeated, then the viewed area <br> contains in turn: 1. all labelled points; 2. all <br> visible points; 3. all visible objects; 4. the user- <br> saved view area. <br> Note. In general, OK geometry manages alone <br> the viewed area by displaying all drawn points. |
| :--- | :--- | :--- |

### 4.3. Point commands

$\left.\begin{array}{|l|l|l|}\hline \text { Command } & \text { Meaning } & \text { Comments } \\ \hline \text { 54. Point } & \begin{array}{l}\text { Draw a point } \\ \text { on an object } \\ \text { Draw an } \\ \text { intersection } \\ \text { point }\end{array} & \begin{array}{l}\text { Click on an 'empty' region to draw a 'free' point. } \\ \text { If clicked on an object (e.g. line or circle) the drawn point is } \\ \text { attached to that object. } \\ \text { If clicked near the intersection of two objects the } \\ \text { intersection point is constructed. (See also Intersections for } \\ \text { the set of all intersections of two objects.) } \\ \text { If clicked near the intersection of three or more objects, } \\ \text { you are asked to sequentially select two of the objects - } \\ \text { the new point is the intersection of these two objects. } \\ \text { Note. Consider the Safe (points) state. See the Safe points } \\ \text { command. }\end{array} \\ \hline \text { 55. Point on object } & \begin{array}{l}\text { Draw a point } \\ \text { that is linked to } \\ \text { an object }\end{array} & \begin{array}{l}\text { Click on an object (e.g. line or circle). The new point will be } \\ \text { linked to that object. } \\ \text { Note. Consider the Safe (points) state. See the Safe points } \\ \text { command. }\end{array} \\ \hline \text { 56. Intersection } & \begin{array}{l}\text { Construct a } \\ \text { point as an } \\ \text { intersection of } \\ 2 \text { objects }\end{array} & \begin{array}{l}\text { Click in sequence on two objects (e.g. line, circle, polyline, } \\ \text { conic). The intersection point of the two objects (more } \\ \text { precisely - the nearest intersection point to the clicked } \\ \text { position) will be constructed. } \\ \text { Note. If one of the object is a conic, the other one should } \\ \text { be a line. }\end{array} \\ \text { with fixed } \\ \text { coordinates }\end{array} \quad \begin{array}{l}\text { Declare a point } \\ \text { points also in observation analysis. Thus they are of little } \\ \text { value in observation }\end{array}\right\}$

| 58. Safe points | Set the modality of treatment of line segments and arcs. <br> (See 2.6.) | If Safe intersection is ON then line segments, segments of polylines, and circular arcs objects are interpreted as baselines or base-circles in intersection operation and in dragging operations. When the Safe intersection option is ON, the Safe indicator in the upper right part of the display turns to cyan colour. <br> Example. If a point is defined as the intersection of two segments (with Safe intersection ON), then the point is defined as the intersection of the respective base-lines. Such intersection point exists even if the two segments (when dragged) do not intersect each other. <br> Example. If a point is linked to a line segment (with Safe intersection ON), the point can be dragged along the base-line of the segment. <br> If Safe intersection is OFF then intersections and dragging behave as intuitively expected. When the Safe intersection option is OFF, the Safe indicator in the upper right part of the display is not coloured. <br> The Safe ON option is more robust. It may be preferred in far-away dragging, in implicit constructions. |
| :---: | :---: | :---: |
| 59. Midpoint | Construct the midpoint between two points <br> Construct the midpoint of a segment or arc <br> Construct the point at halflength of a polyline <br> Construct midpoint of each segment of a polyline <br> (See 2.3) | Pick two points or a segment, arc or a polyline. Eventually, choose appropriate option. |

$\left.\begin{array}{|l|l|l|}\hline \text { 60. Divide ratio } & \begin{array}{l}\text { Construct the } \\ \text { point along an } \\ \text { arc, a segment } \\ \text { or a polyline } \\ \text { that divides it } \\ \text { in length in a } \\ \text { given ratio } \\ \text { Construct } \\ \text { points on a } \\ \text { polyline that } \\ \text { divide each of } \\ \text { its segment in } \\ \text { a given ratio } \\ \text { (See 2.3) }\end{array} & \begin{array}{l}\text { Pick two points or a segment, arc or a polyline. Eventually, } \\ \text { choose appropriate option. } \\ \text { Works like Midpoint, the only difference is that you have to } \\ \text { enter the desired ratio of the length of the obtained } \\ \text { segments. } \\ \text { The ratio a/b or a:b divides a segment in ratio a:b. For } \\ \text { example, dividing the segment AB with ratio 1:2 we obtain } \\ \text { a point C that lies at 1/3 of distance from A to B. }\end{array} \\ \hline \text { 61. Centre of circle } & \begin{array}{l}\text { Construct the } \\ \text { lentre of a } \\ \text { circle (circular } \\ \text { arc) }\end{array} & \begin{array}{l}\text { Construct the } \\ \text { point on a } \\ \text { given object } \\ \text { that is nearest } \\ \text { to a given point } \\ \text { (See 2.3) }\end{array}\end{array} \begin{array}{l}\text { Pick: 1. an object (line, line segment, polyline, arc); 2. a } \\ \text { point. } \\ \text { The command constructs a point on the object (1) that is } \\ \text { nearest to the point (2). } \\ \text { Note. If the selected object (1) is a polyline, then you can } \\ \text { obtain the nearest point on the shown segment, on each } \\ \text { segment, or on the whole polyline. }\end{array}\right\}$

| 64. Triangle centres | Construct the 4 <br> classical <br> triangle <br> centres | In the form that appears, select the type of triangle centre. <br> Then, either pick a triangle (as a polyline) or the three <br> triangle's vertices. |
| :--- | :--- | :--- |
| 65. Harmonic <br> conjugate | Construct the <br> harmonic <br> conjugate of a <br> point with <br> respect to two <br> given reference <br> points. | Select first the two reference points. Then select the point <br> to conjugate wrt. them. <br> Note. If the point to conjugate is not collinear with the <br> reference points, it is replaced with its projection onto the <br> line through reference points. |
| 66. Vertices | Treat all <br> vertices of a <br> polyline as <br> points. | Some commands produce a polyline with no marked <br> vertices on it. Use this command to obtain all vertices of <br> such polylines. |
| 67. Conic points | Construct the <br> characteristic <br> points of a <br> conic. | Pick a conic or 5 points that determine the considered <br> conic. The command constructs a set of points. The number <br> and the meaning of the constructed points depends on the <br> type of the considered conic. <br> If the conic is an ellipse then 7 points are constructed: the <br> centre, the two focuses and the endpoints of both axes. |


|  |  | If the conic is a hyperbola then 5 points are generated: the <br> focuses, the centre and the two intersections of the <br> hyperbola with the axis. |
| :--- | :--- | :--- |
| 70. Intersections/ |  |  |
| Farthest |  |  |
| intersection | Construct the <br> intersection <br> point of two <br> object that is <br> farthest to a <br> given point | Select two objects (circles, lines, arcs, polylines, conic) and <br> a point. <br> Note. If one of the object is a conic, the other one should <br> be a line. |
| 69. Intersections/ <br> Nearest <br> intersection | Construct all <br> intersection <br> points of two <br> objects | Select two objects (circles, lines, arcs, polylines, conic). The <br> command constructs the set of all intersection points of the <br> two objects. <br> intersection <br> point of two <br> object that is <br> nearest to a <br> given point <br> nete. If one of the object is a conic, the other one should |
| First select two objects (circles, lines, arcs, polylines, conic) <br> and a point. <br> Note. If one of the object is a conic, the other one should <br> be a line. |  |  |

$\left.\begin{array}{|l|l|l|}\hline \begin{array}{l}\text { 71. Grids/Uniform } \\ \text { divide }\end{array} & \begin{array}{l}\text { Construct a } \\ \text { pattern of } \\ \text { equally spaced } \\ \text { points along an } \\ \text { object or } \\ \text { between 2 } \\ \text { points } \\ \text { (See 2.3) }\end{array} & \begin{array}{l}\text { Select: 1. two points or a line segment, arc, or polyline; } 2 . \\ \text { the number of segments (not points!) to be generated. } \\ \text { A pattern of the specified number (2) of points is } \\ \text { constructed along the object (1). } \\ \text { If the selected object (1) is a polyline, then the number of } \\ \text { segments refers to either path segments along the polyline, } \\ \text { along each segment of a polyline or a long the picjed } \\ \text { segment of the polyline. }\end{array} \\ \hline \text { 72. Grids/Rectangular } \\ \text { grid } & \begin{array}{l}\text { Construct a } \\ \text { grid of points } \\ \text { in the form of a } \\ \text { parallelogram }\end{array} & \begin{array}{l}\text { Using this command you can, for example, divide a line } \\ \text { segment into a specified number of pieces. }\end{array} \\ \text { corner of the grid; the upper left corner of the grid; 4. the the lower left corner of the grid; 2. the lower right } \\ \text { of points in the vertical direction. }\end{array}\right\}$

### 4.4. Line commands

| Command | Meaning | Comments |
| :--- | :--- | :--- |
| 74. Line: 2 points | Construct a line through <br> two points | Point to two points. You can point either to an <br> existent or nonexistent point - in the later case a new <br> point (free or linked) is implicitly defined. |
| 75. Line: point + <br> angle | Construct a line with a <br> fixed inclination | Construct a line segment <br> given its endpoints |
| Point to the two edges of a line segment to be <br> constructed. You can point either to existent or <br> nonexistent point - in the later case a new point (free <br> or linked) is implicitly defined. <br> Note. If the Safe mode is ON, the displayed segment <br> behaves as a line. |  |  |


| 77. Polyline | Construct a polygonal line given its vertices | Click sequentially to the vertices of the polygonal line. You can point either to existent or nonexistent points - in the later case new points (free or linked) are implicitly defined. <br> To draw a new polygon line, click again the command Polyline. <br> To draw a closed polygon just end at the start vertex. There is no indication whether a polyline is closed or whether it should be interpreted as a polygonal region. <br> Note. The construction tends to avoid selfintersections. In particular the quadrilaterals are automatically modified to avoid self-intersections. <br> Note. If the Safe mode is ON, the displayed segments of polyline in some situations behave as a lines. |
| :---: | :---: | :---: |
| 78. Parallel line | Construct a line that is parallel to a given line and passing through a given point | Pick a line and a point or vice versa. <br> Note. To obtain a perpendicular line press Alt button in editor's toolbar (or use the mouse-wheel). |
| 79. Perpendicular line | Construct a line that is perpendicular to a given line and passing through a given point | Pick a line and a point or vice versa. <br> Note. To obtain a parallel line click the Alt button in editor's toolbar (or use the mouse-wheel).. |
| 80. Bisector | Construct the bisector of a line segment | Pick the endpoints of a segment or pick a line segment. |
| 81. Angle bisector | Construct the angle bisector <br> See 2.5. | Select two lines. The bisector of the resulting angle is displayed. To obtain the alternative solution click the Alt button in editor's toolbar (or use the mousewheel). <br> Or, pick three points: 1. a point on a ray; 2. the vertex of the angle; 3. a point on the second ray. |
| 82. Line: 2 objects | Construct a line determined by two selected objects in terms of passing through points, tangent to circles or conics, being parallel or perpendicular to a | Select two objects (points, lines, circles, conics). A line that touches the two objects is drawn. <br> Note. This command comprises most of the lines commands. The resulting line is determined by two objects and the relations 'passing through', 'parallel with', 'tangent to', 'radical axis'. <br> Note. Usually there are two or more alternative |


| line, a radical axis of two <br> circles | solutions. To obtain them click the Alt button in <br> editor's toolbar or use the mouse wheel. <br> With this command and the mouse wheel to construct <br> lines in a variety of context, for example: <br> a line through 2 points; <br> a line tangent to 2 circles; <br> a line parallel or perpendicular to a given line and <br> touching a given circle; <br> the radical axis of two circles; <br> a line passing through a point and touching a conic; |
| :--- | :--- | :--- |

### 4.5. Circle commands

| Command | Meaning | Comments |
| :--- | :--- | :--- |
| 83. Circle: <br> centre + <br> point | Construct a circle given <br> its centre and a radial <br> point. | Point to: 1. the centre of the circle; 2. a point on the <br> circumference. |
| You can point either to an existent or nonexistent point |  |  |
| -in the later case a new point (free or linked) is |  |  |
| implicitly defined. |  |  |,


| centre + radius | its centre and the numerical value of the radius. | the radius of the circle (either by writing the value on the information line, picking a previously set parameter, or pointing to a circle). |
| :---: | :---: | :---: |
| 85. Compass | Construct a circle given its radius (as the length of a line segment or the radius of another circle) and its centre. | Set: 1. the radius of a circle by pointing to a line segment, to the endpoints of a line segment or a circle; 2. the centre of the circle to be constructed. |
| 86. Apollonius circle | Given two points construct the locus of points $X$ with a given ratio of distances from $X$ to the two given points. | Pick: 1. the first point; 2. a point on the Apollonius circle (to be constructed); 3. the second point. <br> Any point $X$ on the resulting Apollonius circle has the property that the ratio of distance from $X$ and (1) and from $X$ and (3) is the same as the ratio of distance from (2) and (1) and from (2) and (3). |
| 87. Circle: <br> centre + object | Construct a circle given its centre and an object (circle, line or point) that touches the circle. See 2.5 | Pick: 1. the centre of the circle, 2. the object that touches the circle. The object can be a circle, a line or a point. <br> Note. Usually there are more alternative solutions. To obtain them click the Alt button in editor's toolbar or use the mouse-wheel. |
| 88. Circle: <br> radius + 2 <br> objects | Construct a circle with a given radius and 2 objects (circles, lines or points) that touch the circle. <br> See 2.5. | Set: 1. the first touching object; 2. the second touching object; 3 . the numeric value of the radius of the circle (on the information line, click OK to confirm). <br> Note. Usually there are more alternative solutions. To obtain them click the Alt button in editor's toolbar or use the mouse-wheel. |
| 89. Circle: | Construct a circle that | Set: 1. the first touching object; 2. the second touching |


| central angle +2 objects | 'touches' two given objects (circles, lines or points) so that the circle radiuses to the touchpoints form a desired angle. | object; 3. the angle (in degrees) between the radiuses to the touch-points (on the information line, click OK to confirm). <br> Note. Usually there are more alternative solutions. To obtain them click the Alt button in editor's toolbar or use the mouse-wheel. |
| :---: | :---: | :---: |
| 90. Circle: 3 objects | Construct a circle that 'touches' three given objects (circles, lines or points). <br> See 2.5 | Pick the three objects (points, lines, circles) that the circle is supposed to touch. <br> Note. Usually there are more alternative solutions. To obtain them click the Alt button in editor's toolbar or use the mouse-wheel. <br> With this command you can, for example: draw a circle that touches two lines and passes through a given point; circumscribe a circle to a triangle (point to three vertices); inscribe a circle to a triangle (point to three sides), draw a circle that touches three given circles; and so on ... |
| 91. Arc 3 pts | Construct a circular arc given its endpoints and a point inside the arc | Pick: 1. an endpoint of the arc; 2. a point inside the arc; 3. the other endpoint of the arc. |
| 92. Arc centre | Construct a circular arc given its centre and two endpoints | Pick: 1. the centre of the arc; 2. the first endpoint of the arc (considering the positive orientation of the arc); 3. the approximate second endpoint of the arc (considering the positive orientation of the arc). |
| 93. Indicate arc | Construct an arc indicating a circle | Pick: 1. the centre of the arc; 2.the approximate centre of the indicated arc . |


| 94. Conic 5 <br> points | Draw a conic given 5 <br> points on it. | Pick 5 points on the conic to be drawn. <br> Note. Other constructions of conics are available in <br> menu Advanced\|Shapes|Other. |
| :--- | :--- | :--- |

### 4.6. Number commands

| Command | Meaning | Comments |
| :---: | :---: | :---: |
| 95. Distance to object | Find the distance from a point to an object (another point, a line, a circle) | Set: 1 . the point from where the distance is measured; 2. the object to which the distance is measured (can be a point, a line or a circle); 3 . the name of the parameter with the measured value. |
| 96. Radius of circle | Find the radius of a circle or circular | Pick a circle or arc and declare the name of the parameter. |
| 97. Angle size | Find the size of an angle | Note. The angles are always expressed in degrees. <br> Note. The measured size of the angle can be positive or negative, depending on the orientation of the angle. <br> First option: 1. pick a point on the angle's ray; 2. pick the angle vertex; 3 . pick a point on the other angle's ray; 4. write the name of the parameter with the size of the angle. <br> Second option: 1. pick a base line of a ray; 2. pick the base line of the other ray; 3 . write the name of the parameter that will contain the size of the angle between base lines. <br> Note. The measured angle between baselines is always smaller or equal to the right angle. |
| 98. Length, circumferen ce | Find the circumference of a circle <br> Find the length of a line segment or circular arc <br> Find the length of a polyline <br> Find the length of a segment of a polyline | Point to a circle, a line segment or a polyline. If you select a polyline you need to specify whether you the length refers to the pointed segment or to the whole polyline. <br> Then write the name of the parameter that will contain the length/circumference. |

\(\left.$$
\begin{array}{|l|l|l|}\hline \text { 99. Area } & \begin{array}{l}\text { Find the area of a } \\
\text { given circle } \\
\text { Find the area } \\
\text { enclosed in a polyline }\end{array} & \begin{array}{l}\text { Select a circle or a polyline. Then write the name of the } \\
\text { parameter that will contain the measured area. } \\
\text { Note. The polyline should be closed and without self- } \\
\text { intersections, otherwise the resulting area is not valid. }\end{array} \\
\hline \begin{array}{r}\text { 100. Ratio of } \\
\text { segments }\end{array} & \begin{array}{l}\text { Find the ratio of the } \\
\text { lengths of two line } \\
\text { segments }\end{array} & \begin{array}{l}\text { Either select two existing line segments (they can be a part } \\
\text { of a polyline) or pick the 4 endpoints of the line segments. } \\
\text { Then write the name of the parameter that will contain the } \\
\text { ratio of the lengths of the segments. }\end{array} \\
\hline \begin{array}{l}\text { 101. Value, } \\
\text { Expression }\end{array} & \begin{array}{l}\text { Calculate the numeric } \\
\text { value or the value of } \\
\text { an arithmetic } \\
\text { expression }\end{array} & \begin{array}{l}\text { Note the input box that appears in the information line and } \\
\text { the OK button near it. } \\
\text { In the input box write a numeric value for a parameter. } \\
\text { Then write the name of the parameter (variable). Such } \\
\text { numerical parameters can be dynamically changed (see the } \\
\text { command Action/Drag Parameter). } \\
\text { In the box you can write as well an arithmetic expression, } \\
\text { which may contain mathematical functions and previously } \\
\text { defined numeric parameters. Such expressions cannot be } \\
\text { 'dragged'. }\end{array}
$$ <br>
To insert a numeric parameter in the expressin just click on <br>
the parameter in the Transformations \& Parameters section <br>
(on the left pane). Note that the names of the parameters <br>
are written in brackets. <br>
Expressions should comply with the usual syntax rules. <br>
Expressions may contain mathematical functions (see the <br>

list below). For the power sign you can use either ^ or **.\end{array}\right\}\)| Here are some examples: |
| :--- |

$\left.\begin{array}{|c|l|l|}\hline \begin{array}{r}\text { 102. Compose } \\ \text { conditions }\end{array} & \text { Compose conditions } & \begin{array}{l}\text { There are several commands for setting geometric } \\ \text { conditions (e.g. whether three points are collinear), see the } \\ \text { command Check property in section 4.8. Sometimes it is } \\ \text { necessary to compose previously set geometric conditions. } \\ \text { The only two allowed operations are logical conjunction } \\ \text { (and) and logical disjunction (or). } \\ \text { Conditions are composed in the box that appears in the }\end{array} \\ \text { information line. To insert a previously set condition just } \\ \text { click on the condition. As mentioned, in the expression may } \\ \text { contain only: conditions, and, or, parentheses. }\end{array}\right\}$

Here is a list of the functions that can be used in numeric expressions:

| ABS $(x)$ | absolute value of $x$ |
| :--- | :--- |
| ACOS $(x)$ | arcus cosines of $x$ (degrees) |
| ASIN( $x$ ) | arcus sinus of $x$, in degrees |
| ATAN $(y, x)$ | arcus tangens of $y / x$, in degrees |
| ATANG $(x)$ | arcus tangens of $x$, in degrees |
| CEIL $(x)$ | smallest integer that is bigger or <br> equal to $x$ |
| $\operatorname{COS}(x)$ | cosine of $x$ (degrees) |
| DEG( $x)$ | express radians in degrees |
| $\operatorname{EXP}(x)$ | exponent function |
| FLOOR $(x)$ | largest integer that is smaller or <br> equal to $x$ |
| FROUND( $x)$ | round $x$ to the nearest integer |
| $\operatorname{GCD}(x, y)$ | greatest common divisor of $x$ and <br> $y$ |


| HYPOT $(x, y)$ | length of hypotenuse in right <br> triangle with sides $x$ and $y$ |
| :--- | :--- |
| IFIX $(x)$ | integer part of $x$ |
| LCM $(x, y)$ | least common multiple of $x$ and $y$ |
| LOG $(x)$ | natural logarithm of $x$ |
| MAX $(x, y)$ | the biggest of $x$ and $y$ |
| MIN $(x, y)$ | the smallest of $x$ and $y$ |
| PI | the constant pi |
| RAD $(x)$ | $x$ degrees expressed in radians |
| RAND $(x, y)$ | a random number between $x$ and <br> $y$ |
| REM $(x, y)$ | remainder of $x$ when divided by $y$ |
| RND | random number between 0 and 1 |
| SIGN $(x)$ | sign of $x$, can be $-1,0$ or 1 |
| SIN( $x)$ | sine of $x$ (degrees) |
| SQRT $(x)$ | square root of $x$ |
| TAN $(x)$ | tangens of $x$ (degrees) |

### 4.7. Transformations commands

Transformations in OK Geometry are treated as explicit objects. You can apply a transformation immediately after you define it, or later.

Suppose you want to define a translation that moves a given point A to a given point B. After you select the command Transformation/Trans/ation you are asked to pick the point A and the point B. A transformation called Tran_AB is generated and can be immediately applied to an object. The transformation is 'visible' in Transformation \& Parameters list. It is possible to change the default name of the transformation. You can refer to and apply the transformation later using the command Transform/Transform object.
$\left.\begin{array}{|l|l|l|}\hline \text { Command } & \text { Meaning } & \text { Comments } \\ \hline \text { 104. Translation } & \begin{array}{l}\text { Declare a translation and } \\ \text { apply the translation or } \\ \text { its inverse } \\ \text { Define a glide translation } \\ \text { and apply the translation } \\ \text { or its inverse }\end{array} & \begin{array}{l}\text { First select the a point (origin) and the point where } \\ \text { you want the origin to be translated. } \\ \text { Note. Press Alt button at this point if you want to } \\ \text { declare a glide translation. } \\ \text { Then pick the object you want to apply the (glide) } \\ \text { translation. } \\ \text { Symmetry }\end{array} \\ \hline \text { 105. Reflection } & \begin{array}{l}\text { Declare a reflection } \\ \text { symmetry (wrt. a line) } \\ \text { and apply it }\end{array} & \begin{array}{l}\text { Pick a line or two points of the line of reflection. } \\ \text { inverse transformation. }\end{array} \\ \hline \text { Then pick the object to be transformed. point to apply the } \\ \text { symmetry and apply it }\end{array} \quad \begin{array}{l}\text { Pick the centre of rotation symmetry. Then pick the } \\ \text { object to be transformed. }\end{array}\right\}$

| 109. Similarity | Declare a similarity <br> transformation and <br> apply the transformation <br> or its inverse <br> Declare a glide similarity <br> transformation and <br> apply the transformation <br> or its inverse | Select: 1. the first origin point; 2. the image of first <br> origin point; 3. the second origin point; 4. the image <br> of the second origin point. <br> Note. Press Alt button at this point if you want to <br> declare a glide similarity. <br> Then pick the object you want to apply the (glide) <br> similarity. <br> Note. Press Alt button at this point to apply the <br> inverse transformation. |
| :--- | :--- | :--- |
| 110. Projectivity | Declare a projectivity <br> that maps four given <br> points to four given <br> points | Select an origin point and then its image point. <br> Repear this for all four origin and image points. <br> Then pick the object you want to apply the <br> projectivity. |
| 111. Inversion wrt. | Invert a point, line or <br> circle wrt. a circle | Note. Press Alt button at this point to apply the <br> inverse transformation. |
| Pick: 1. a circle; 2. the object (point, line, circle). |  |  |
| The inversion of the point wrt. the selected circle is |  |  |
| constructed. |  |  |


| 115. Other/ Multiply object by grid | Position in various ways copies of a given object along a grid of points | In this command it is necessary to consider and object $O$ with two selected points related to it: the guide point $G$ and the reference point $R$. <br> Furthermore, we shall refer to the variable point of the grid as $S$. This command positions along a grid copies of object, one for each point of the grid. For each point $S$ on the grid a copy of the transformed object O is generated under the composition of: <br> an optional dilatation with centre G and factor \|GS|/|GR|; <br> an optional rotation that carries the line $R G$ to line RS; <br> an optional translation that carries G to S . <br> To execute the command select: 1. the object O to position along the grid; 2 . the guide point G ; 3 . the reference R; 4. any grid-point (setPoint) on the grid. |
| :---: | :---: | :---: |
| 116. Multiply along polyline | Position geometrically similar copies of a given object along each segment of a polyline | This command positions similarity copies of an object along the sides of a polyline. The similarity transformation brings two given reference points of the object to the endpoints of respective sides of the polyline. <br> Select: 1. the object to position along the grid; 2. the 1st reference point of the object (a point); 3. the 2nd reference point of the object (a point); 4. the polyline along which to position the object. |
| 117. Repeat transformation | Apply recursively a number of times a transformation to an object | Select: 1. the transformation; 2. the number of repetitions; 3 . the object to which to apply recursively the transformation. <br> A sequence of object will appear, each transformed from the previous one with the given transformation. |

### 4.8. Advanced commands

| Command | Meaning | Comments |
| :---: | :---: | :---: |
| 118. Shapes | Quick <br> construction <br> of various <br> commonly <br> used shapes <br> (isosceles <br> triangle, <br> rectangle, <br> regular n-gon <br> and many <br> others) | With this command you can easily construct commonly used shapes by positioning or approximately positioning some vertices of the shape to be constructed. <br> In the form that appears first choose the type of shape (triangle, quadrilateral, other) and then the specific shape. Then follow the instructions on the information line. <br> Note that: <br> The input vertices of the shape should follow a positive orientation (for some shapes this is immaterial). <br> Not all vertices of the constructed shape are shown. Make them accessible with the command Point/Vertices. <br> By default the vertices of the constructed shapes are not labelled. You can label them with commands Action/Labels/Auto label or Action/Labels/Label all. |
| 119. Check property | The <br> command comprises a series of test of geometric objects (collinearity of points, congruence of triangles and many others). | This set of commands contains checks for a wide range of geometric properties. The main use of these checks is related to the command Advanced/Implicit construction. Using implicit construction it may be possible to modify a given construction so that a geometric property becomes fulfilled. <br> In the form that appears when the command is clicked first choose the type of property you want to check (position, equivalence, shape). And select a property from the displayed list. Then follow the instructions on the information line. |
| 120. Locus | Displays the locus of a constructed point as another point moves along an object. | Select: 1. the constructed point to be followed (should depend on the position of point (2); 2. the point to be moved; 3. the object (circle, arc, line, line segment, polyline) along which (2) is supposed to move. <br> The resulting object is a curve, which represents the trail of (1) as (2) moves along (3). <br> Note. It is not required that the point (2) lies on the object (3). <br> Note. If the resulting locus appears to be related to an underlying common object (line, circle or conic), you can generate that object too. |


| 121. Implicit locus | Obtain a hint for the locus of points for which a given constructionrelated parameter is fixed | The command tries to suggest a possible locus of a specified variable point in the construction, so that a specified construction-related parameter has a desired value. Only points, lines, circles, and conics are eventually suggested as locuses. <br> Note. With this command it is, eventually, possible to obtain just a hint for the sought locus. The hint should be always critically evaluated. <br> Note. The presence of implicitly constructed locus reduces the reliability of observational results. |
| :---: | :---: | :---: |
| 122. Implicit construction | Find the position of a point so that a condition is satisfied <br> Obtain a hint for the locus of points for which a given condition is satisfied <br> Modify up to three objects in the construction so that condition is satisfied | There are two versions for this command. In both cases the program tries to modify one or more objects in a construction so that a condition is fulfilled, i.e. so that one or more properties hold. <br> In the simple version of the command the program tries to find the position of a given point so that a previously set condition is fulfilled. The position of a point may be arbitrary or the point may be linked to an object. <br> In case that several points on the plane fulfil a given condition, OK Geometry may provide a hint for the locus of such points. <br> Make the initial construction. <br> Using the command Advanced/Check property find whether the target condition is satisfied (e.g. whether two objects coincide, whether three points are collinear). If it is false and you would like to make it true by moving a point in a construction, proceed with the find command. <br> A form (for simple version of the command) appears. Choose one of the three options for implicit construction. The first two options deal with situations where we expect the solution in form of a single point (either on an object or on the plane). The last options deals with situations where we expect the solution in form of a line, circle, conic. <br> After you fill the form, click OK and pick the point to be moved. If you have check-marked the constraint option for the variable point (i.e. an object on which to restrict the search), click the object to which the moving point is linked. <br> Check whether the desired position of the moved point has been found, i.e. whether the sought condition is true. <br> The advanced version of this command allows modifying up to three objects (points, lines, circles) or numeric parameters. Furthermore you can: |


|  |  | Define a restriction for each object. <br> If the positions of the objects to be modified are almost correct, <br> then select 'local optimisation'. <br> Note. The presence of implicitly constructed objects reduces <br> the reliability of observational results. |
| :--- | :--- | :--- |
| 123. Optimisation | Modify one <br> or more <br> objects in a <br> construction <br> so that the <br> value of a <br> parameter is <br> optimised | Optimisation procedure is similar to implicit construction. <br> Instead of a condition to fulfil the aim is to optimise a scalar <br> value. We optimise the scalar value by automatically changing <br> the position of one or more objects. <br> Make the initial construction. <br> Compute (as a numeric parameter the value to be optimised. <br> Proceed as in the case of Advanced/Implicit objects. <br> Note. The presence of optimisation procedures reduces the <br> reliability of observational results. |
| 124. Make macro | Declare a <br> sequence of <br> commands as <br> a new <br> command <br> Save macros <br> to be used in <br> other <br> constructions. | First we need to carefully make a construction in which some <br> (dependent) objects are generated from some initial objects. <br> Then click the command Advanced/Make macro and proceed <br> as follows: <br> Fill the form that describes the macro. In particular, choose the <br> types of the input object(s) and the type of output object(s). <br> Adjacent to the selected types of input objects you may write <br> comments on how to select objects. Press the Continue button. <br> Pick the input and the output objects in the appropriate order. <br> If the macro is successfully constructed you can save it to a file <br> (to be used in other occasions). In any case you can use the <br> generated macro in the current session with the command <br> Advanced/Execute macro. |
| 125. Get macro | 126. Execute <br> macro | Read one or <br> more macros <br> saved in from <br> previous <br> sessions. <br> macro a |
| Select a macro and execute it as any other command. |  |  |

## 5. Examples

The examples in this section illustrate the use of some commands of OK Geometry Sketch editor. The Sketch editor differs from common programs of dynamic geometry. Its main aim is to enable quick and effective constructions of geometric configurations to be submitted for further analysis in OK Geometry. In this way the user is focused on analysing and proving properties.

Usually there are many ways how to work out a construction task in OK Geometry Sketch editor. In examples below we show just one way - in order to illustrate a command or a construction strategy.

### 5.1. Midpoints in a quadrilateral

This example demonstrates how certain commands are applied in different ways to a polyline.

| Task: | In an arbitrary quadrilateral $A B C D$ mark the midpoints $\mathrm{E}, \mathrm{F}, \mathrm{G}, \mathrm{H}$ of the four sides. Which interesting properties of the obtained configuration can you spot? |  |
| :---: | :---: | :---: |
| File: | OkEditEx_1.p |  |
| Strategy: | Draw the quadrilateral as a shape. <br> Draw the midpoints using the Midp | nt command |
| Procedure/S tep | Sketch editor command | Comment |
| 1 | Shapes\|Quadrilateral | Pick 4 vertices. You obtain an unlabelled quadrilateral. |
| 2 | Point\|Midpoint | Point to the quadrilateral (on one of the edges). In the form that appears and select the option 'On each segment' to obtain all midpoints at once. |


| 3 | Action\|Labels|Label all | Label all generated points. The order of labels <br> follows the order of construction of points. |
| :--- | :--- | :--- |
| Suggestions <br> for the <br> analysis | According to the construction we are interested in properties that are invariant under <br> the cyclic permutation of vertices: ABCD EFGH. (See OkEditEx1a.pro for an elaboration.) <br> What can you say about the shape of the quadrilateral EFGH? <br> How does the its area relate to the area of ABCD? (See OkEditEx1b.pro for an <br> elaboration.) |  |

### 5.2. Inscribing a circle

This example demonstrates the use of complex construction commands to produce configurations to be analysed.


| 2 | Circle ${ }^{\text {CircleC obj }}$ | Construct a circle tangent (from inside) to the outmost circle. <br> Pick the centre (T). <br> Pick the outer circle. <br> Click on Alt button until the shown new circle is the desired one. |
| :---: | :---: | :---: |
| 3 | Circle ${ }^{\text {Line }} 2$ pts | Construct the line through the points S and T . |
| 4 | Circle\|Line 3 obj | Construct the circle tangent to the three drawn objects: <br> Pick the outer circle, the inner circle, and the drawn line (any order of picking will do). Click the Alt key until the new shown circle is the desired one. |
| 5 | Point\|Centre of circle Point | Pick the newly constructed circle to obtain its centre. <br> Point to the position of the relevant points. |
| 6 | Action\|Labels|Clear all labels <br> Action\|Labels|Label vertex <br> Action\|Labels|Label auto | After you set the label A skip to Label auto.Then pick the points in the desired order to obtain labels A,B,C,D. <br> Work in a similar way for labels S,T,U. |
| 7 | Action ${ }^{\text {Colours and styles }}$ | Select first the desired colour, width and line style and then select the line or circle to be modified. <br> Repeat this step as desired. |
| Suggestion s for the analysis | - How would one do the construction using Euclidean tools? The analysis reveals lots of properties, but we want to know the position of the point $C$ on line $A B$ and the position of $D$ right above the point $C$. <br> - To obtain the position of $C$ use the Filter search and look for ratios $A C$ and $B C$ (in the Filter search write: $A C \& \& B C$ ). <br> - To obtain the radius CD look for ratios of distances containing CD. Using such data you will perhaps come to a beautiful 3 -square constuction shown below. (First draw the squares that determine the red dotted line and the point $C$. Then draw the square that determines the blue dotted line and the point D. See OkEditEx_2a.p .) |  |

How to prove the ratios you will (hopefully) find among the relations? A possible way to
find out the relations between the radiusses of the three circles and the distance AC is by
considering right triangles related to the mentioned quantities. In fact, using Pytagoras
theorem on triangles TCD and SCD and doing a bit of algebra we can prove the correctness
of the construction.

### 5.3. Isosceles triangles on top of a triangle

This example demonstrates how to define and apply a transformation.

| Task: | On each of the sides of a given <br> triangle ABC draw three similar <br> isosceles triangles. <br> Study the obtained configuration. |
| :--- | :--- |
| Oile: | OkEditEx_3.p |


| Strategy: | Draw a triangle $A B C$. On one of the sides place an isosceles triangle. <br> Using similitude transformations construct copies of the isosceles triangles on the two other sides. |  |
| :---: | :---: | :---: |
| Procedure/ <br> Step | Sketch editor command | Comment |
| 1 | Shape\|Triangle <br> Action\|Labels|Label all | Draw a triangle. Label all the vertices, they are labelled as A,B,C. |
| 2 | Shape\|Isosceles <br> Action\| Labels|Label all | On the side BC draw an isosceles triangle: Point to $C$, to $B$ and then approximately where you want $D$ to be. |
| 3 | Transform\|Similarity | First we shall define a similarity that maps $B C$ to <br> CA. We shall apply this similarity immediately to the isosceles triangle and to the point $D$. <br> Pick the point $B$, then to $C$ (the desired image of $B$ ). <br> Select the point $C$, then to $A$ (the desired image of C). <br> Name the obtained transformation as Simil_BC_CA. Since we shall immediately apply it, click Transform. <br> Immediately apply the transformation to the isosceles triangle (point on it), and to point D (point it). <br> Analogously construct a triangle on the side $A B$. |
| 4 | Action\|Auto label <br> Action\|Colours and styles | Label properly the vertices. <br> Apply the desired attributes to lines and points. |
| Suggestion s for the analysis | According to the construction we are interested in properties that are invariant under the cyclic permutation of vertices: ABC DEF. <br> The configuration has some very interesting properties. Detect them with the command Observe. |  |

### 5.4. A cyclic tangential quadrilateral

The following example demonstrates the usage of implicit construction of a point.

| Task: | Construct a quadrilateral that is both cyclic (i.e. its vertices lay on a circle) and tangential (i.e. all sides are tangent to a circle). <br> Study the obtained quadrilateral. <br> Given the inscribed circle and the points E,F,G, how to construct the point H , so that the resulting quadrilateral $A B C D$ is cyclic? |  |
| :---: | :---: | :---: |
| File: | OkEditEx_4.p |  |
| Strategy: | Starting from an (in)circle we pick 4 poin circles at these points. The resulting cyclic. In fact, as we draw the (out)cir it. <br> Using implicit construction command that the point $D$ will lay on the outer <br> An analysis of the resulting configurat construct the point H given the (in)cir | nts, $\mathrm{E}, \mathrm{F}, \mathrm{G}, \mathrm{H}$ on it and draw the tangent lines to the adrilateral $A B C D$ is tangential but, in general, not through $A, B, C$ and observe that $D$ does not lay on <br> f OK Geometry we position H on the (in)circle so cle, $A B C D$ will then be cyclic and tangential. <br> n will (hopefully) provide a hypothesis how to and the points E,F,G. |
| Procedure/ Step | Sketch editor command | Comment |
| 1 | Circle\|Circle <br> Point\|Point | Draw the incircle. <br> Position the points E,F,G,H on it. |
| 2 | Line\|Line 2obj | To draw the tangent line to the incircle at E pick the point $E$ and the incircle. In the same way construct the tangents to the incircle at $\mathrm{F}, \mathrm{G}$, and H . |
| 3 | Point $\mid$ Point Action\|Hide | Construct the intersection points $A, B, C, D$. Hide the four tangent lines. |


|  | Line\|Polyline | Draw the quadrilateral $A B C D$. |
| :---: | :---: | :---: |
| 4 | Circle ${ }^{\text {Circle 3obj }}$ | Pick the points $A, B, C$. The outcircle through A, B, and C appears. |
| 5 | Advanced\|Check property|Position|PointOnObject | Pick the point $D$ and then the outcircle. Name the resulting condition 'positionD'. <br> The condition positionD appears (as False) in the list of parameters. <br> (In the next step we shall instruct OK Geometry to move the point H along the incircle so that the condition positionD becomes True.) |
| 6 | Advanced\|Implicit construction | In the form that appears check the option Move a point along some object since we want to position H on the 'incircle' (object) so that the condition positionD (that D lays on the 'outcircle') will be met. Leave the other options as suggested. Click the OK button in the form. <br> Pick the point to be modified, i.e. the point $H$. <br> Finally, pick the constraint object, i.e. the 'incircle', along which H will be positioned. <br> (Now H and D should appear in a desired position.) |
| Suggestion s for the analysis | According to the construction we are interested in properties that are invariant under the cyclic permutation of vertices: ABCD EFGH. The analysis will gain additional evidence an additional cyclic permutation $A B C D F H$ is specified. <br> The cyclic tangential quadrilateral have many indeed interesting properties. Study them with the Observe command. <br> How to construct the point H in Euclidean steps? Note that the incircle and the points $\mathrm{S}, \mathrm{E}, \mathrm{F}, \mathrm{G}, \mathrm{B}$ and C may be considered as given. Among the properties listed by OK Geometry we look for some property that relates the given objects with the point H . <br> One of such relations is shown on Figure 7. It is easy to show that the two emphasized triangles are similar isosceles triangles. Making use of this fact the construction of the point H is rather obvious. <br> Note. The presence of implicitly constructed objects reduces the reliability of observational results. |  |

Position a point to make a condition true (Simple).

1. Fill this form (OK):
2. Select a condition (OK).
3. Pick the point.
4. If checked, pick a constraint (point, line) for the point.

Afterwards you may use Fix to accelerate work.

- Point on object (condition + object)
$C$ Point on plane (condition)
$C$ A hint for locus of points (condition)
$\bigcirc$ A hint for locus of points (parameter to fix)
$\sqrt{ }$ Fast method (local, less reliable)
OK Advanced Cancel

Figure 7

### 5.5. The angular size of a line segment

The following trivial example demonstrates the implicit construction of the locus of points.

| Task: | Given a line segment AB, we want to <br> find instances of points P, from <br> which the segment AB is visible <br> under an angle of a fixed size. In <br> other words, we are looking for the <br> locus of points for P where the angle <br> APB is constant. |
| :--- | :--- | :--- |
| File: | OkEditEx_5.p |
| Strategy: | Given the segment AB we pick an arbitrary point $P$ and measure the angle APB. Then we <br> ask OK Geometry to find some instances of positions of point $P$ in which the angle APB has |


|  | the desired value. |  |
| :---: | :---: | :---: |
| Procedure/ <br> Step | Sketch editor command | Comment |
| 1 | Point <br> Action\|Label vertex <br> Number\|Angle | Draw the vertices $A, B$ and the point $P$. <br> Then measure the angle APB. Name the measured value as AngleAPB. |
| 2 | Advanced\|Implicit locus | Fill the form as shown on the left: <br> Since we are looking for the locus for P where a parameter (i.e. AngleAPB) is fixed, fill the form as shown on the left and click OK. <br> After the form is filled pick the point $P$ (to variable object). <br> (A circle - the circumcircle of APB appears, see Figure 8.) |
|  | Important note. As is evident from this trivial example, OK Geometry possibly provides only a hint for the sought locus of points. In our example the correct locus is the arc BPA (since it refers to the size of the oriented angle), and not the whole circle. Depending on the situation it may happen that OK Geometry misses the locus, provides an 'extended' locus or even gives a deceiving hint. It is just a hint that requires additional checks. |  |



Figure 8

### 5.6. The pedal points

The following example demonstrates the usage of implicit construction.

| Task: | The orthogonal projections of a point $P$ to the base lines of the three sides of a triangle $A B C$ are called the pedal points of $P$ wrt. the triangle $A B C$. The pedal points are the vertices of the pedal triangle of point $P$ wrt. the triangle $A B C$. <br> Does it happen that the pedal points are collinear? When? |  |
| :---: | :---: | :---: |
| File: | OkEditEx_6.p |  |
| Strategy: | Given a triangle $A B C$ and an arbitrary <br> With the command Implicit objects we condition that their pedal points $\mathrm{D}, \mathrm{E}$, | oint $P$ we construct the pedal points $D, E, F$. find several instances of points $P$ that satisfy the are collinear. |
| Procedure/ <br> Step | Sketch editor command | Comment |
| 1 | Point\|Point <br> Line\|Line 2pts <br> Line\|Polyline | Draw the vertices $A, B, C$, and the point $P$. <br> Construct the lines $A B, B C, C A$. <br> For reference draw the triangle $A B C$ as polyline. <br> Note: We need the lines since we project $P$ to the base lines of the triangle's sides. |
| 2 | Point\|Nearest point | For each base line (!) find the projection of $P$ to the line as nearest line to $P$. The easiest way is to determine $D$ as the point on the baseline $B C$ that is nearest to $P$. In the same way construct the points $E$ and $F$. <br> Note. The line segments PD, PE, PF are shown only for reference. |


| 3 | Advanced\|Check property|Position|CollinearP | Check whether D, E, and F are collinear. Name the result as CollinearDEF. |
| :---: | :---: | :---: |
| 4 | Advanced\|Implicit construction | Since we are looking for the locus of points of $P$ that satisfy a given condition (i.e. CollinearDEF), fill the form as shown of the left. <br> Then click OK and pick the point $P$ (to be moved around). <br> (The sought locus appears, see Figure 8.) |
|  | Important note. OK Geometry possibly provides only a hint for the sought locus of points. In our example the found locus is indeed correct (by Simson's theorem). However, depending on the situation it may happen that OK Geometry misses the locus, provides an 'extended' locus or even gives a deceiving hint. The result this always requires additional checks. |  |
| Suggestion s for the analysis | Given the obtained hint, it is easy to provide a more reliable check that the points on the suggested locus satisfy the considered condition. <br> You can use OK Geometry to prove the theorem (start with a new construction where $P$ lies on the circumcircle of $A B C$ and analyse the situation. The proof is not trivial. Try also to investigate, where should $P$ lie in order that the pedal triangle has a fixed area (proceed as in example 5.5). |  |



Figure 9

### 5.7. The Fermat point of a triangle

This example will demonstrate how to generate a hypothesis for the solution of geometric optimisation tasks.

| Task: | For a given triangle $A B C$ find the point $D$ for which the sum of the distances from the vertices $A, B, C$ is minimal. |  |
| :---: | :---: | :---: |
| File: | OkEditEx_7.p |  |
| Strategy: | Given a triangle $A B C$ and an arbitrary and calculate the sum of these distan around to a position where the sum | oint $D$ we measure the lengths of $A D, B D$, and $C D$ . Then we let OK Geometry to move the point $D$ distances to the vertices is minimal. |
| Procedure/ Step | Sketch editor command | Comment |
| 1 | Shape\|Triangle <br> Action \|Label all <br> Point[Point | Draw a triangle. <br> Label the vertices $A, B, C$. <br> Draw and arbitrary point D . |
| 2 | Number\|Distance to object | Measure the distance from $D$ to $A$. Point to the two points. Name the parameter DistanceAD. <br> In the same way measure DistanceBD and DistanceCD. <br> In the next step we shall calculate the sum of the three measured distances. |
| 3 | Number\|Value, expr | Click first on DistanceAD. In the information row appears [DistanceAD]. Then click the + key on the |


|  |  | keyboard. Click on the DistanceBD. Repeat this also for DistanceCD In the information row appears <br> [DistanceAD] + [DistanceBD] +[DistanceCD]. <br> Click the red coloured OK to enter this expression. Name the result Sum_AD_BD_CD. <br> In the next step we let OK Geometry move around the point $D$ so that Sum_AD_BD_CD attains a minimal value. |
| :---: | :---: | :---: |
| 4 | Advanced\|Optimisation | In the form that appears: <br> Select the minimum option (Optimise for), uncheck the Constraints option, since $D$ is not restricted to any object (line, circle, conic). Prefer the Fast method. In case the results are not as expected try with the option unchecked. <br> Select the parameter to be optimised, i.e. Sum_AD_BD_CD. Then click OK in the information line. <br> Pick the point to be moved around (D). <br> The point $D$ is appears on new position (Figure 10). <br> Note, if one of the angles is bigger than 120 deg. the point D is positioned behind one of the vertices. |
| Suggestion s for the analysis | The point $D$ assumes the position of the Fermat point (if all the interior angles are less than 120 degrees). <br> Use Observe to find the characteristic property of Fermat's point. Note that the property should be invariant under the cyclic permutation of labels ABC. |  |
|  | Note. The presence of optimisation methods reduces the reliability of observational results. |  |



Figure 10

### 5.8. A minimal triangle

This example will demonstrate how to generate hypothesis for the solution of a complex optimisation tasks. The solution can be afterwards analysed with Observe. From the analysis a proof can be inferred.

| Task: | In a given triangle ABC inscribe the <br> triangle DEF with minimal perimeter. |
| :--- | :--- | :--- |
|  |  |


| Strategy: | Given is a triangle $A B C$. We draw another triangle DEF - not necessary inscribed in $A B C$ - and measure its perimeter. <br> Then by (automatically) moving the points $D, E, F$ along the respective sides of the triangle $A B C O K$ Geometry will adjust the points D,E,F to obtain the triangle with minimal perimeter. <br> A subsequent analysis will reveal the properties of the optimised triangle DEF. From the observed properties a proof may be inferred. |  |
| :---: | :---: | :---: |
| Procedure/ Step | Sketch editor command | Comment |
| 1 | Advanced\|Shapes|Triangle|Triangle Action|Label all | Draw a triangle. Label the vertices $\mathrm{A}, \mathrm{B}, \mathrm{C}$. <br> Draw also another triangle, label the vertices D,E,F. You may place these vertices along the sides of $B C, C A, A B$, but this is not necessary. |
| 2 | Number\|Length,circumference | Measure the perimeter of triangle DEF. Name the value as PerimDEF. <br> Note. In the next step we shall move D,E,F along $B C, C A, A B$ so that PerimDEF will attain a minimal value. |
| 3 | Advanced\|Optimisation | In the form that appears choose the Advanced button. Check the entries as shown in Figure 11. <br> In particular: <br> Select the parameter to minimise (PerimDEF). <br> Select the minimum option. <br> Select 3 variable objects since we shall move three points: D,E,F. <br> Check the Constraints option since the three points shall move along the sides of the triangle ABC. <br> Use Fast method, if possible. <br> To confirm the options click OK in the form. <br> Click the point E , then the restriction (the pointed |


|  |  | segment CA of $A B C$ C. <br> Click the point $F$, then the restriction (the pointed <br> segment $A B$ of $A B C)$. <br> The point $D, E, F$ are given the calculated optimal <br> position. |
| :--- | :--- | :--- |
|  | In the Treatment section set the Invariant cycle to ABC DEF. <br> Inspect the properties of the found solution. The properties may lead you to a proper <br> construction and also to the proof of the correctness of your construction. |  |


| 76 Optimisation mode |  |  |  |  $\square$ |
| :---: | :---: | :---: | :---: | :---: |
| Optimise a parameter by positioning objects(Advanced). <br> 1. Fill this form (OK). <br> 2. Select the parameter to optimise (OK). <br> For each object to be positioned: <br> 3. Pick the object. <br> 4. If checked, pick a constraint (point, line, ...) for the object. <br> Afterwards you may use Fix to accelerate work. |  |  |  |  |
| Optimise for <br> c minimum <br> $\bigcirc$ maximum <br> $C$ target value | Variable objects <br> C 1 object <br> C 2 objects <br> - 3 objects |  | Other options Constraints for objects Fast method (local, less reliable) <br> $\Gamma$ Objects are numeric parameters |  |
| Value |  |  |  |  |
|  | OK | Simple | Cancel |  |

Figure 11


Figure 12

### 5.9. Making a macro

This simple example will demonstrate how to make a macro.

| Task: | Given two points $A, B$ we would like to construct a 'left bulged' path between them like shown right. All the involved segments should measure one third of the distance between $A$ and $B$. The angles between segments should measure 120 and 60 degrees. <br> Since we need to connect several points in this way we shall make a macro. |  |
| :---: | :---: | :---: |
| File: | OkEditEx_9.p |  |
| Strategy: | Given the points $A, B$ we shall first draw draw and equilateral triangle on the <br> At this point we shall define the macr be the input data and what exactly th points $A$ and $B$. We want the output d on the way. | points at each third between them. Then we shall ddle part and draw the desired path as a polyline. <br> It is very important to think in advance what will output data. In our case the input data are the ta to be the resulting polyline and the three points |
| Procedure/ Step | Sketch editor command | Comment |
| 1 | Point\|Uniform divide <br> Shape\|Equilateral triangle | Divide a line segment in three equal parts. <br> Click on points $A, B$, and write 3 (the number of parts) in the information line. Confirm by clicking OK in the information line. <br> Construct an equilateral triangle using the generated points in the middle part of the segment.. |
| 2 | Action\|Label all Line|Polyline | It is important to label the points to be part of the output. <br> Draw the polyline for output by clicking the vertices in appropriate order. |


| 3 | Macros\|Make macro | Fill the form as shown in Figure 13. <br> After you click OK Geometry you need to point the <br> input and then the output objects in the same <br> order as specified in the form. <br> In the final form that appears you may choose to <br> use the constructed macro only with this file or to <br> store it separately and use it elsewhere (with the <br> Get macro command). <br> Note. The macro is not part of the construction. <br> The next time you open the construction the <br> macro will not be available unless you store it <br> separately and read it separately with <br> Advanced\|Get macro command. |
| :--- | :--- | :--- |
|  | Advanced\|Execute macro | The constructed macro is a new command <br> available in the form that appears. |



Figure 13


[^0]:    ${ }^{1}$ Do not confuse the Alt button with the Alt key on the keyboard.

