

OK Geometry Basic

Part I

Reference for OK Geometry Basic (v.18.1)

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1 What is OK Geometry?

OK Geometry is a tool for analysing geometric constructions. Given a dynamic geometric construction, OK Geometry detects patterns as well as certain properties of the construction. OK Geometry can also help organise the information found.

OK Geometry was initially intended for students that often find it difficult to cope with geometry exercises such as construction tasks or tasks that require building up a proof. Solving such tasks requires an appropriate organisation of the properties of the configuration. A necessary step in solving many geometry tasks is thus to be aware of the properties of a configuration. OK Geometry can help students to overcome this step.

For teachers, this program can be useful for creating exercises and for presenting geometric properties and formal proofs in electronic or written format.

For recreational mathematicians interested in classical geometry, this program can be a useful tool for analysing geometric configurations, in particular those related to triangle geometry. OK Geometry contains a huge database of characteristic points, lines, circles, and conics that are used in analysing objects with regard to a given triangle.

You can think of OK Geometry as a pair of geometric spectacles. Using sophisticated numerical and non-numerical methods, OK Geometry detects potential properties of dynamic geometric constructions (e.g. that certain points are collinear or that certain triangles have the same area). **Students should be aware that one thing is to see (observe) a property, another thing is to know that a property is true, and yet another thing to have an explanation why a property is necessarily true.** OK Geometry helps the student in the first step. OK Geometry does not prove that something is true, it is even possible (though very unlikely) that it makes an 'observation error'. OK Geometry helps to observe a configuration, and this is an important step towards knowing why something in the configuration is true. OK Geometry also helps to relate a given object in a configuration to other objects (Is there anything in this configuration that is related to a given angle?). OK Geometry can also help the user to organise properties, but it does not select which properties are important, interesting, or even novel.

To summarise: OK Geometry helps the user to analyse geometric configurations and to reason about its properties, but it is up to the user to decide which properties are relevant for his purpose and, eventually, to use them in building up a proof.

1.1 The model behind OK Geometry

Planar geometry is about configurations of geometric objects such as points, lines, circles etc. Such objects and configurations can be considered from a naive perspective - formally, however, such objects in geometry are abstract entities, and the properties of configurations are derived from an assumed set of axioms from which a system of definitions and theorems is built. Geometric reasoning requires that geometric objects be represented in some way, whether one considers them as naive or as formal. The representations of geometric objects are usually based on a model, so that not only the objects but also the properties among them can be represented in some way. A model thus helps to visualise and understand properties and can be used to informally verify the validity of claims and hypothesise new properties. Unlike the "paper-and-pencil" way of representing geometric objects, which is based on the

static model, and unlike dynamic geometry software, which is based on the dynamic model, OK Geometry uses the stochastic dynamic model for representing geometric objects.

Before we briefly explain the three models we need to clarify an aspect of geometric constructions. Consider, for example, an equilateral triangle (Figure 1). To construct a general equilateral triangle we start from two of its vertices, say A and B, which can be positioned arbitrarily on the plane. The third vertex of the triangle, say C, has to be positioned so that its distance from A and from B is the same as the distance between A and B. We say that A and B are *non-constructed* or *free* objects, while C is a *constructed* object. The line segments AB, BC, and CA are, obviously, also constructed objects since they are determined by object set previously.

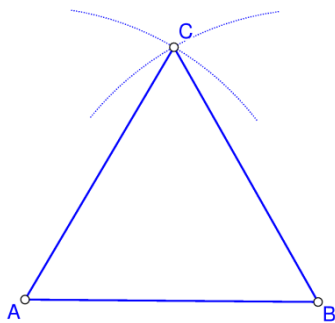


Figure 1

In the **static model** all represented objects, whether free or constructed, are fixed. For example, the plane is represented as a sheet of paper, points by drawn dots on it, etc. The drawn objects are static.

In the **dynamic model** the plane is represented, for example, by a computer display and the objects by dots or sets of dots on it. The free (non-constructed) points can be dragged. The constructed objects cannot be dragged but they change in accordance with the construction and the position of the free points.

The **stochastic dynamic model** is similar to the dynamic model. However, the free (non-constructed) objects are represented by several copies of randomly positioned objects. The constructed objects are represented by several objects constructed from copies of other objects in accordance with the construction. Only one copy of each object is visible, and assumes a reasonably random position (as well as its copies).

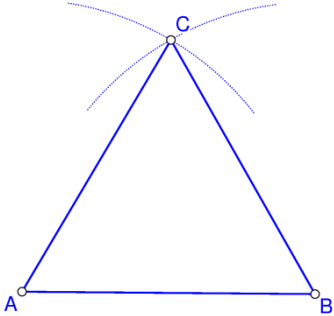
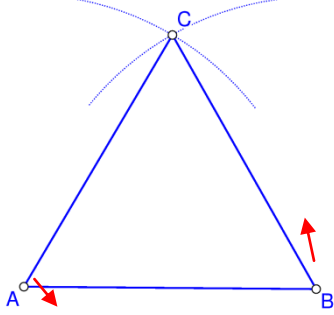
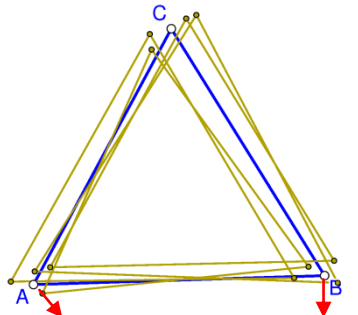
		
<p>The static model. All represented objects are fixed.</p>	<p>The dynamic model. The free points A and B can be dragged. All other objects change accordingly.</p>	<p>The stochastic dynamic model. The free points A and B can be dragged. There are several randomly positioned copies of each object – only one of them is visible.</p>



Figure 2

The three models are illustrated in Figure 2 for the case of an equilateral triangle with given points A and B. The difference between the models becomes clear when a property is checked. In the static model an observed property (based on a measurement) can easily be a matter of chance. The same is true for the dynamic model; however, by dragging, other instances of construction can be created and checked for the property. In the stochastic dynamic model a property is checked simultaneously on all random copies of the object, thus there is no need for dragging when inspecting a property.

1.2 Exact and perturbed view. The blue dot

The main task of OK Geometry is the observation dynamic constructions. When observing, OK Geometry always uses stochastic dynamic representation of objects, so that it considers several copies of perturbed points around each given free point, although it shows only one (perturbed) construction. However, sometimes it is preferred to visualise the construction based on the very given free points. For this reason OK Geometry can visualise both, the exact and the perturbed view.

The blue dot on the left upper corner of the displayed construction serves as an indicator as well as a switch between the exact and the perturbed visualisation. When in exact mode, the blue dot appears as

gray disc with a dot in its centre , while in perturbed mode the blue dot contains some random red dots . The size of the blue dot shows the size of the perturbation area around free points. A **click on the blue dot** changes the modes from one to the other.

Internally, all observations are made in the perturbed mode. On the other hand, all exports of constructions are made in the exact mode, independent of the status of the blue dot.

Note that some constructions cannot be displayed in exact mode.

The power of OK Geometry is, by no means, in the perturbed representation of objects. Sometimes viewing objects in perturbed mode is not desired since points and the whole construction change slightly from time to time. Also, in perturbed mode we cannot draw exact horizontal or vertical lines. On the other hand, in the exact mode, we can draw perfect horizontal and vertical lines (OK Geometry gives a help to do this). There is also a command in Sketch editor to visually align points.

A **right click on the blue dot** allows you to configure some frequently used parameters related to the display of the configuration (e.g. the way of filling areas, font of point labels). In particular, you can also set the level of the displayed help for the construction commands.

1.3 Easy, Basic, and Plus mode of OK Geometry

OK Geometry works in three modes. The **Easy mode** is intended for occasional users - it allows the analysis of geometric constructions created by dynamic geometry software (such as GeoGebra, Cabri, Cabri Express, Sketchometry, Cinderella, C.a.R. (Compass and Ruler), Sketchometry or JGEX). The Easy mode is also intended for solving exercises in deductive reasoning in planar geometry: such exercises can be prepared with OK Geometry (but not in Easy mode). To access the **Basic mode**, click *Configure/Working mode/Basic* on the menu (Figure 3). This document considers only the Basic mode of OK Geometry, which is also part of the Plus mode.

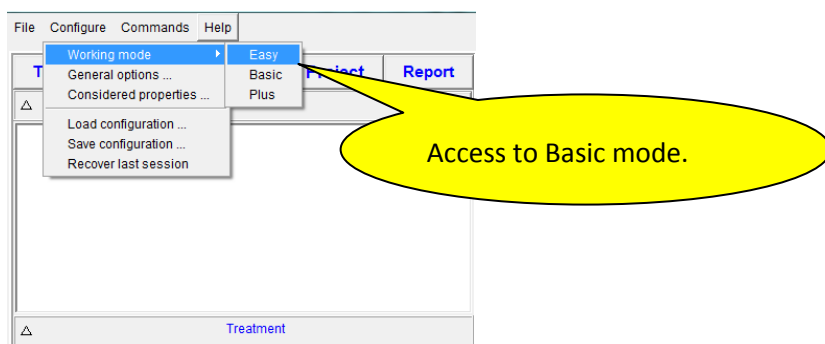


Figure 3

When the title of the main program window is just OK Geometry the program works in Basic mode. Here is a concise list of tasks that OK Geometry can do in the Basic mode:

- import constructions from different dynamic geometry software (DGS), currently for Cabri, Cabri II, Cabri Express, GeoGebra, Cinderella, CaR (Compass and Ruler), Sketchometry, JGEX (Java Geometry Expert);
- make new constructions or edit the imported ones;
- modify constructions in order to achieve certain goals (e.g. automatically move a point in order to obtain a maximal area of a shape in a construction);
- spot patterns and properties in geometric constructions;
- investigate properties related to a specific object in a construction;
- select relevant properties for further elaboration;
- organise the selected information (e.g. to build up a proof);
- present properties of a configuration (or a proof);
- make a report that can be read with common word processors or browsers.

This document describes how to use the OK Geometry in Basic mode.

2 Some terminology conventions

In the text we use the term **(geometric) object** or **entity** to denote any basic geometric object such as point, line, circle, arc, (line) segment, polyline (i.e. a concatenated sequence of line segments). In a broader sense, this list sometimes includes also real numbers (related to geometric quantities) and real valued variables (parameters).

By **(geometric, dynamic) construction** we understand a set of free basic objects (points or real numbers) and a sequence of objects that are obtained from free or previously constructed objects using ‘allowed’ construction methods. The ‘allowed’ construction methods should be reducible to basic Euclidean construction steps.

By **(geometric) configuration** we understand a set of geometric objects positioned in Euclidean plane, usually satisfying specified relations.

We use the term **icon** for a pictorial representation of a geometric property. An icon consists of a configuration with some emphasized elements, together with an associated verbal comment and the underlying dynamic construction. An icon can consist of several **scenes**. Scenes are merely visualisations of an icon in which selected objects are made more or less emphasized and more or less visible.

Several icons are organised in a **project**. A project can depict interesting properties of a geometric configuration or, for example, the steps of a proof of a theorem. The basic (starting) icon of the project is named **Task**.

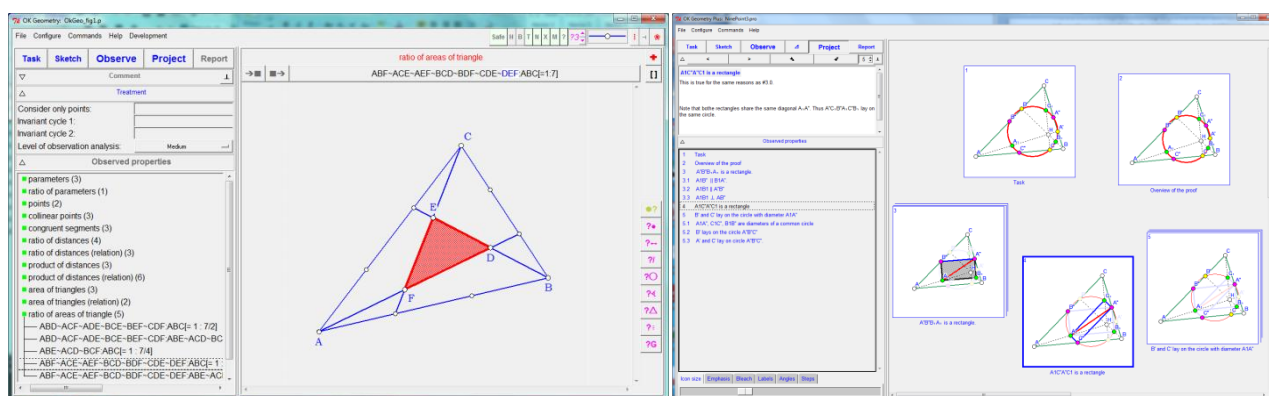


Figure 4

The two central concepts we shall often use are: geometric properties and instances of properties. We shall use the term **(geometric) property** for general properties, e.g. collinearity of points, perpendicularity of lines. By **instance (of a property)** or **relation** we mean a property applied to specific objects in a configuration or construction, e.g. the fact that the points A, B, C in a configuration are collinear is a relation, thus an instance of the property collinearity.

Figure 4 (left) displays a geometric construction (right pane of the left screenshot), its properties and instances of properties (left pane of the left screenshot). A construction can - together with some comment – turned into an icon. Figure 4 (right) displays a project consisting of several icons.

OK Geometry is often used in a very simple fashion (Figure 5), just to study a construction:

1. A dynamic construction is made in Sketch editor, which is a module of OK Geometry, or imported from a dynamic geometry software.

2. The geometric properties of the construction are analysed and studied.

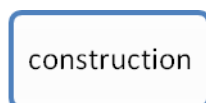


Figure 5

OK Geometry allows also a more complex schema of work (Figure 6). The work, i.e. project, can consist of several commented constructions. A project can be about a theorem and its proof, about properties of a geometric shape, about a family of geometric objects, about a group of related theorems, etc.

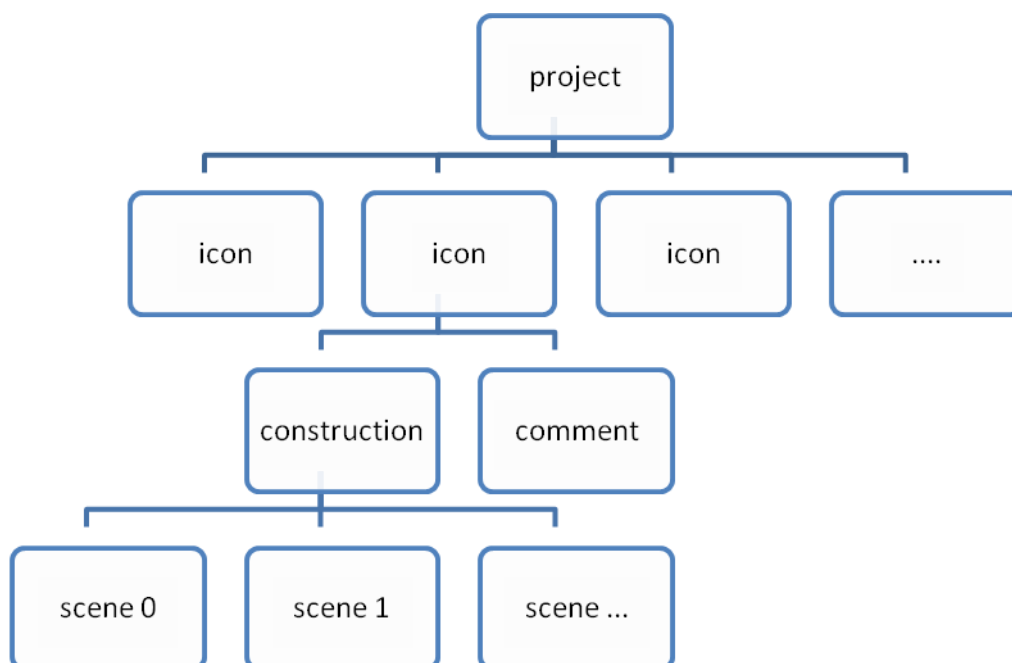


Figure 6

3 The interface of OK Geometry

On the top of the window there is a **menu line** with sets of commands (File, Configure, Commands, and Help). Below the menu line there is the **main menu bar** with buttons for the basic commands. Still below the main menu bar is the **information pane**, which consists of **several sections**. The sections can be minimised, expanded or fully expanded – to change their state click or right-click the button (with up/down oriented triangle) on the left of the title of the section. Finally, on the right there is a **visualisation pane**. The visualisation pane displays a geometric construction (Figure 7) or the icons of a project. The details of the interface in either case differ somewhat, but are rather self evident.

There are several ways to access **help** as you work in OK Geometry. Some commands with help information are located under the rightmost menu item. Additional help information for entries is often available as self-appearing balloons or by **right-clicking on object or entry**. During the **Sketch** operation there is also a **Help** section for editing commands, the section can be turned on or off.

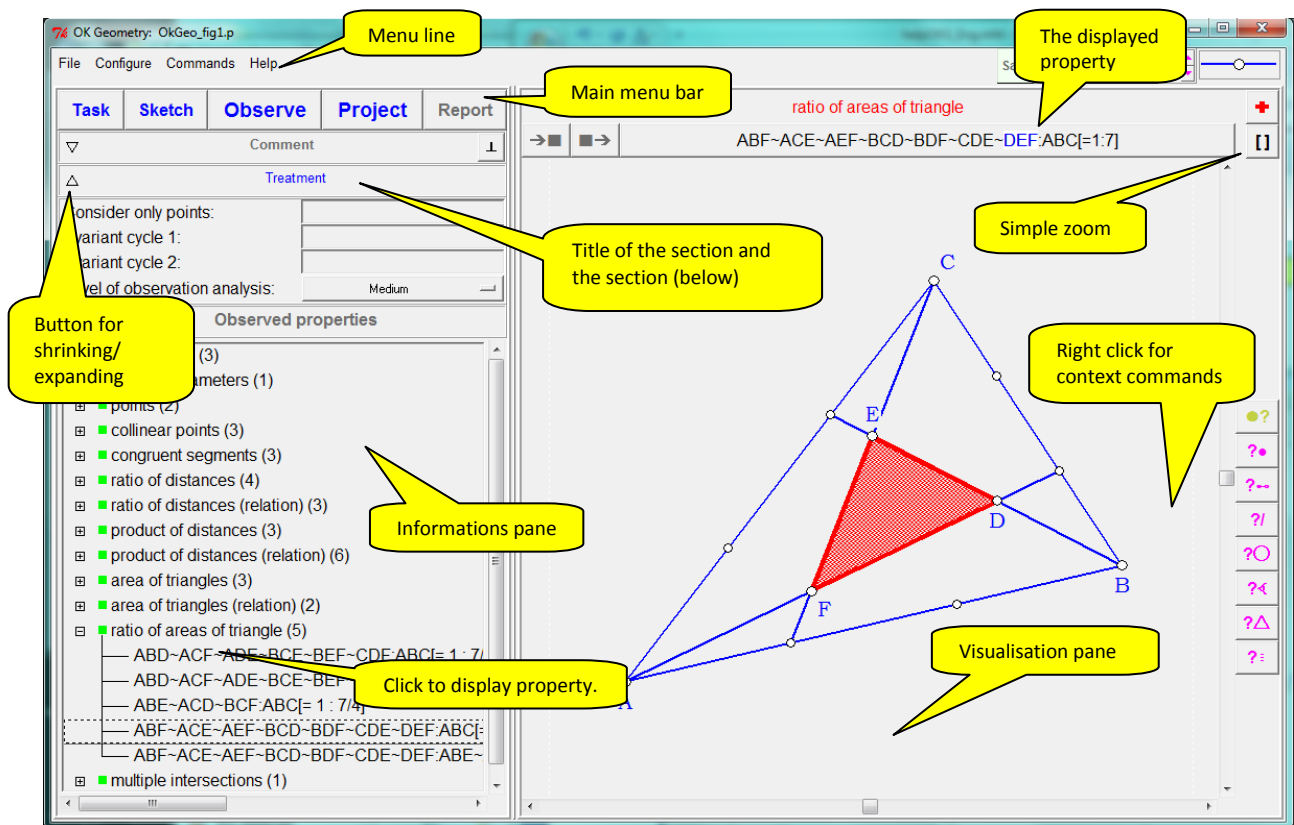


Figure 7

4 Introductory examples

4.1 First example. Detecting properties

We illustrate how to detect geometric properties with Ok Geometry on a very simple situation: Given is a triangle ABC and a point on each of its sides: point D on BC, point E on CA, point F on AB. We shall analyse this configuration.

Although it is possible to draw constructions in OK Geometry, in this introductory example we start with construction files created with dynamic geometry software Cabri, Cabri II, Cabri Express, GeoGebra, C.a.R. (Compass and Ruler), Sketchometry, Cinderella, or JGEX (see Figure 8). You can find the related files (Example1.fig or Example1.ggb, Example1.jc, Example1.zir or Example1.cdy) among the provided examples.

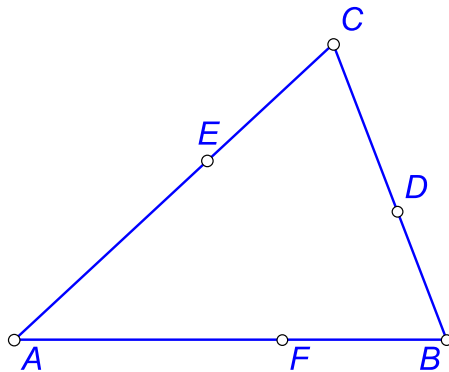


Figure 8

Open OK Geometry in Basic mode and follow these steps:

- Click the button **Task** in the main menu bar and choose your file, e.g. Example1.fig. The construction appears on the right pane.
- Click the button **Observe**. The list of detected properties of the considered construction appears in the left pane in section Properties (see Figure 9).
- Click on a property you that you find interesting, e.g. multiple intersections. The property ‘expands’ and instances of the property appear. Click on any instance of the property to visual it on the right pane. Many of the properties listed are trivial, but you were perhaps unaware of some non-trivial property, e.g. three circles passing through a common point (Figure 10) – a fact known as the Miquel theorem.

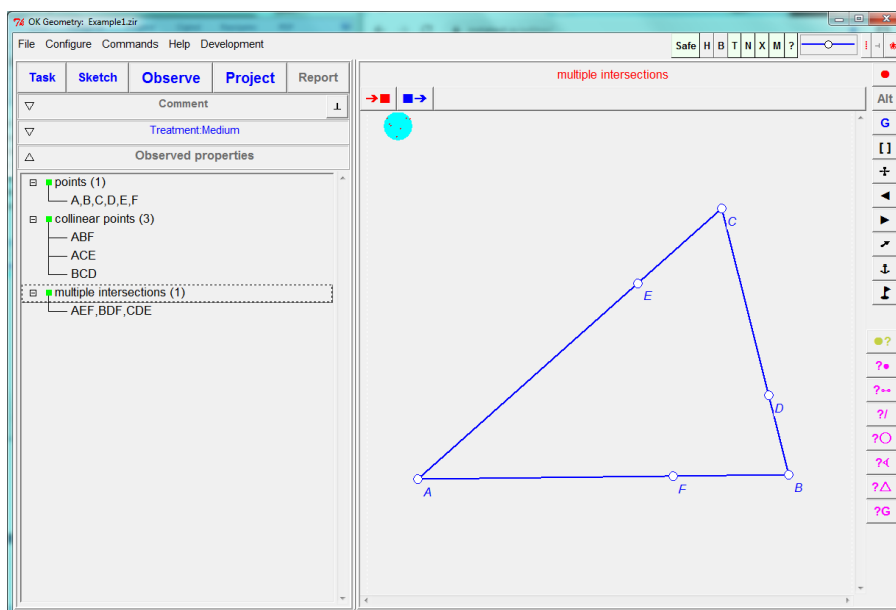


Figure 9

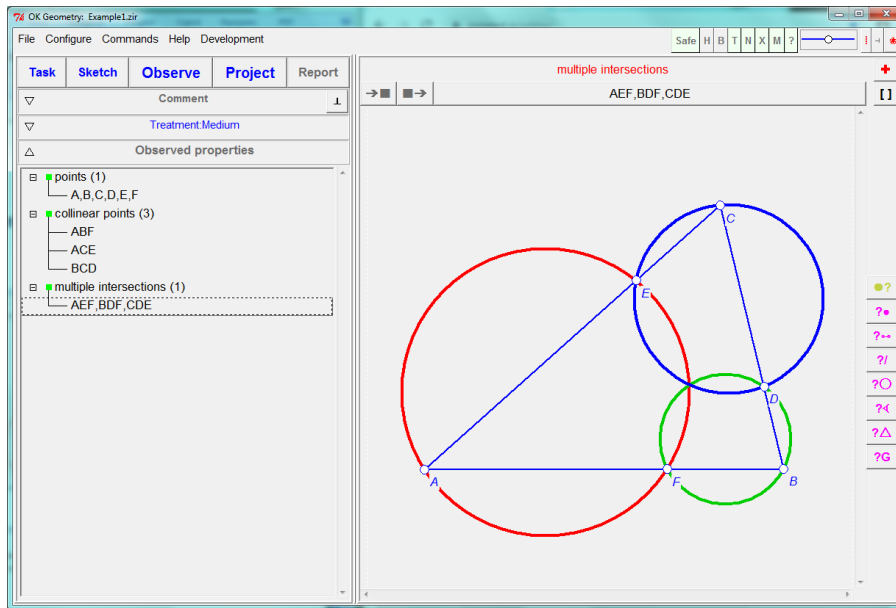


Figure 10

4.2 Second example. Querying objects in a configuration

OK Geometry often detects many properties and many instances of properties. If we are interested only in properties related to some object in the configuration, we can use the query commands. Here is a simple example (see Example4.p).

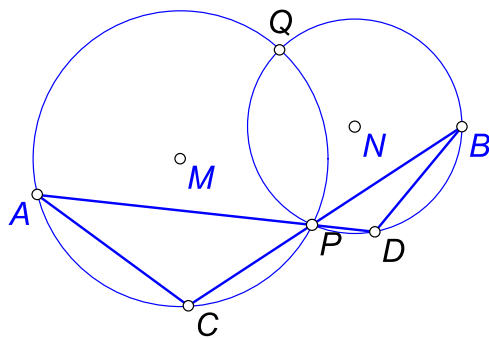


Figure 11

Given are circles $k(M,A)$ and $k(N,B)$ that intersect in points P and Q (Figure 11). Let C be the intersection, other than P , of the line BP and circle $k(M,A)$. Let D be the intersection, other than P , of the line AP and circle $k(N,B)$. On advanced treatment OK Geometry lists more than a hundred instances of properties of this simple configuration, among them 38 angle congruences. Suppose we want to find information about the angle between lines AC and BD . We start from a construction in some dynamic geometry software (Example4.ggb, Example4.jc, Example4.txt, Example4.zir, Example4.cdy) and proceed as in the first example.

1. Apply the command **Task** to import the desired dynamic construction.
2. Apply the command **Observe** to form a list of observed properties.

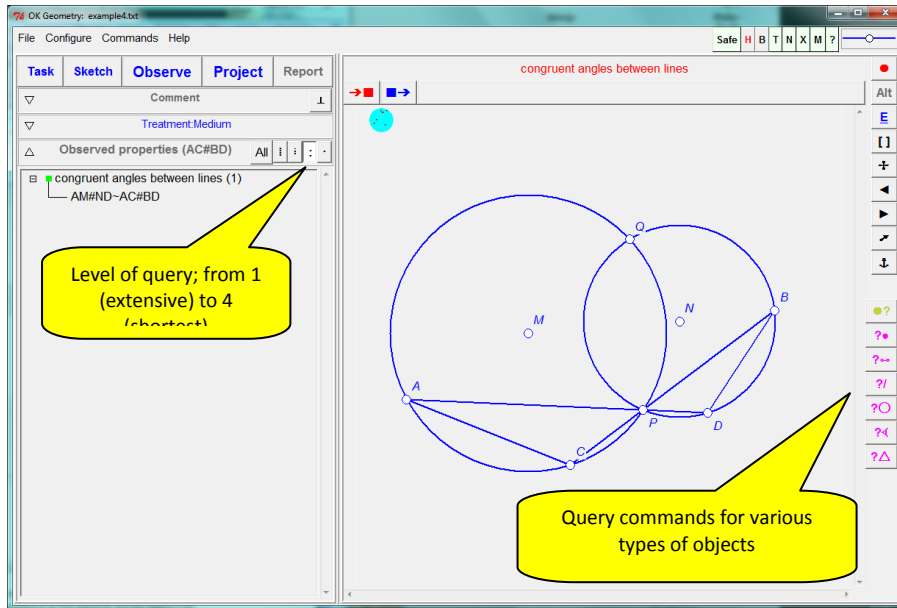


Figure 12

3. On the right taskbar a series of query buttons appears. Click *QueryAngle*, then pick the segment AC (click somewhere in on the segment) and the segment BD (Figure 12).
4. Select the query level (Figure 12). A higher level (less dots) shows more concise information. In our case 1 or 2 (dots) show just one angle congruent to the inspected one (Figure 13), while 3 or 4 dots show 5 angles congruent to the considered one.

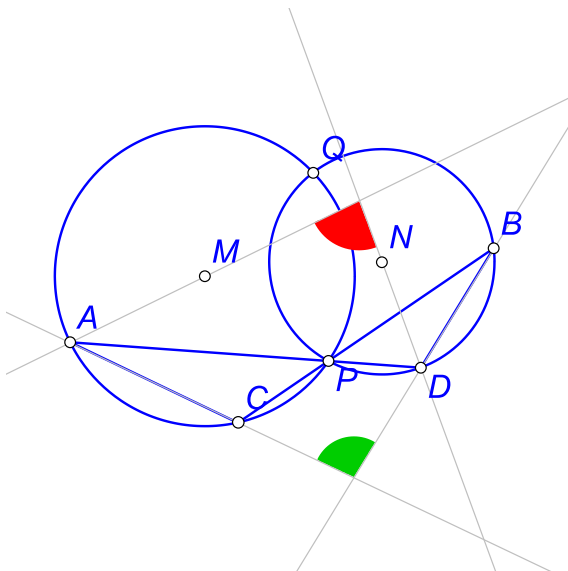


Figure 13

The query commands can be of great help in solving construction tasks.

4.3 Third example. Working on detected properties

Consider the following task: The diagonals of a trapezium split the trapezium into four triangles. Prove that two of the resulting triangles have the same area.

Also in this example we rely on a program for dynamic geometry (Cabri, Cabri Express, Cinderella, C.a.R. (Compass and Ruler), JGEX or GeoGebra). We construct a trapezium (see Figure 14). Note that the vertices should be labelled each with a single letter followed by digits or apostrophs. We also add to the construction the text of the task. We save the construction in file Example2.fig (or Example2.ggb or Example2.cdy). You can use the files in the provided examples.

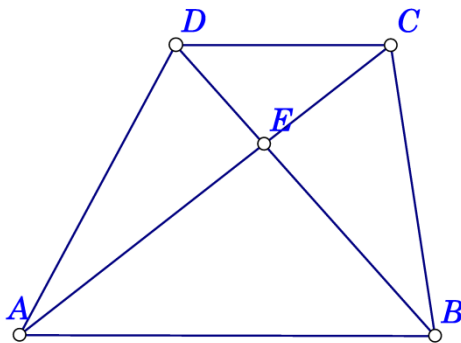


Figure 14

Now open OK Geometry and follow these steps:

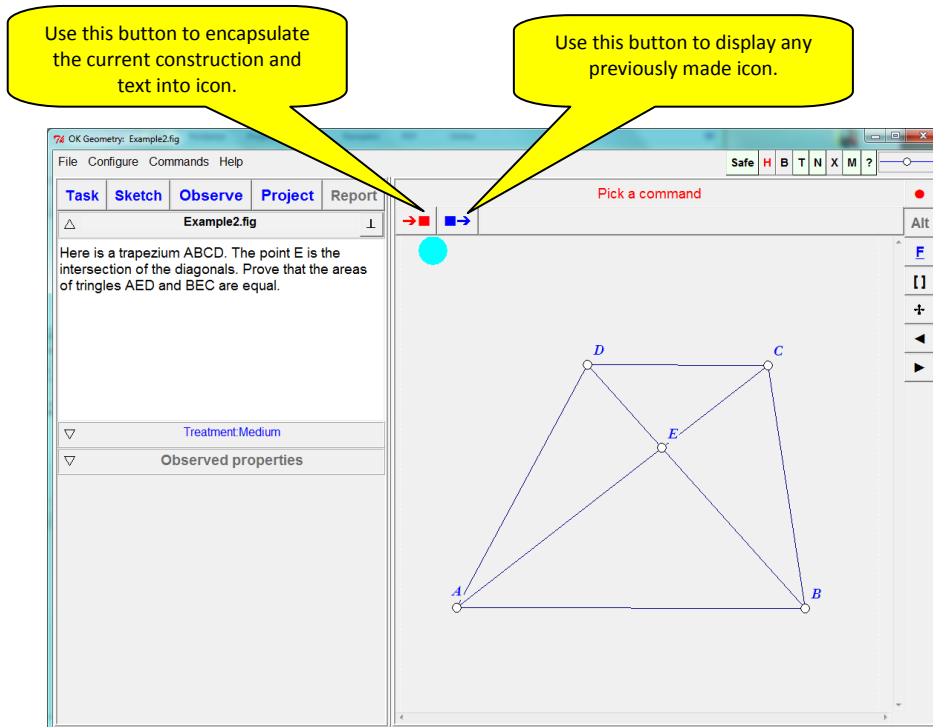


Figure 15

Apply the command **Task** in the main menu bar to import the previously created file with the dynamic construction, e.g. Example2.fig (Figure 15). Since we intend to work on properties of this configuration, we declare the imported construction as the basic icon, the Task. To do this, click on the leftmost icon above the visualisation pane and declare the construction as **1 Task** (Figure 15).

Apply the command **Observe** on the main menu bar to create a list of observed properties of the considered construction. Among the listed properties click on those you find related to your problem, in our case - the areas of triangles. Three instances of the property appear. Click on an instance, e.g. $ABC \sim ABD$, to visualise the instance of property. To visualise a specific object in the relation, e.g. the triangle ABC, click on the name of the object above the drawing pane. (See Figure 16.)

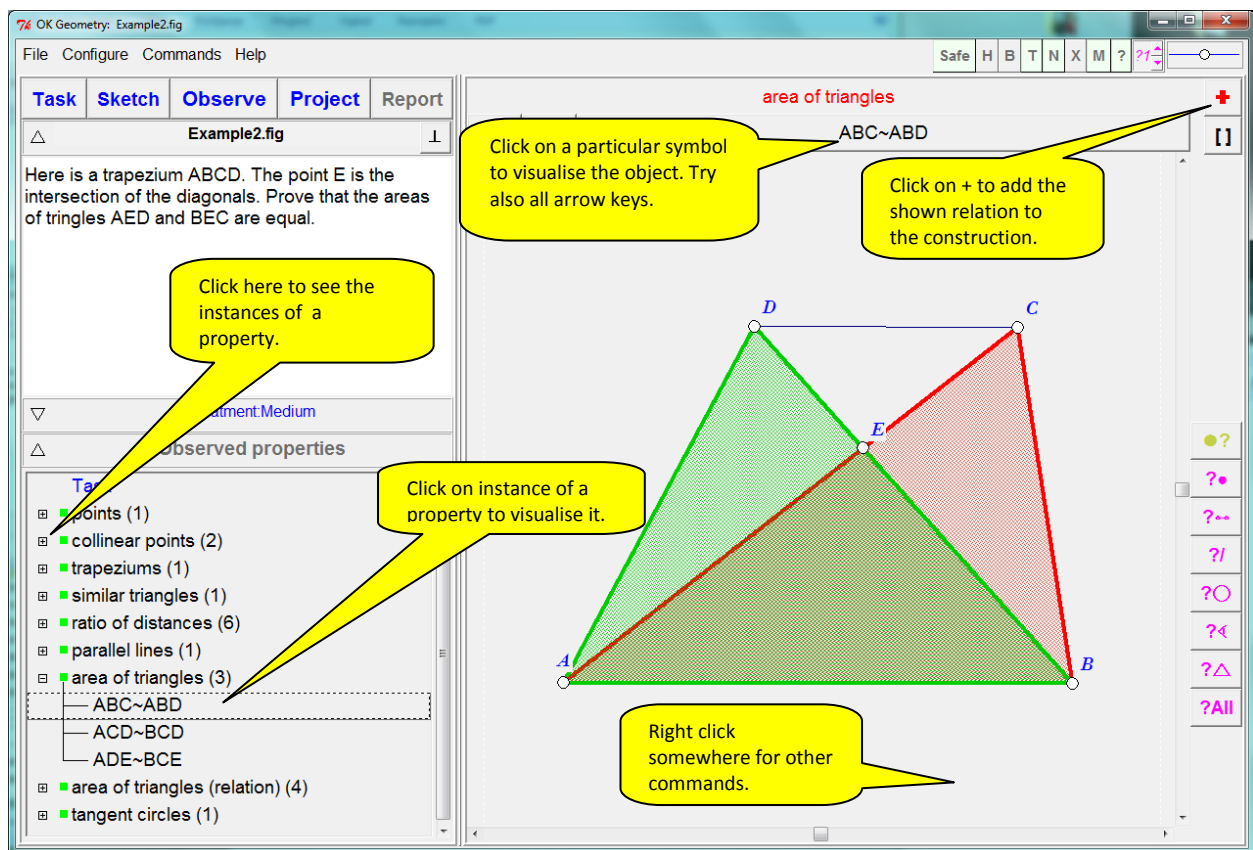


Figure 16

Relations that you find relevant for further consideration can be declared as icons. For example: triangles AED and BEC have the same area, and triangles ABC and ABD have the same areas as well. This is clearly related to the fact that in a trapezium the bases AB and CD are parallel. In order to add the relation to the current construction click on the red '+' icon above the visualisation pane (Figure 15). Usually, the obtained construction requires some simple modifications (see Figure 17):

- select another angle between two lines using the **anchor button** repeatedly;
- hide objects;
- delete objects;
- modify the colour or style of objects;
- emphasise, bleach or make object transparent.

It is not a good idea to drag free points or to re-label the points of the configuration since the icon are (probably) used in relation to the basic task (Task icon).
When the presentation of the relation is as desired, declare it as a **New** icon (or overwrite a previous one) using the $\rightarrow \blacksquare$ button.

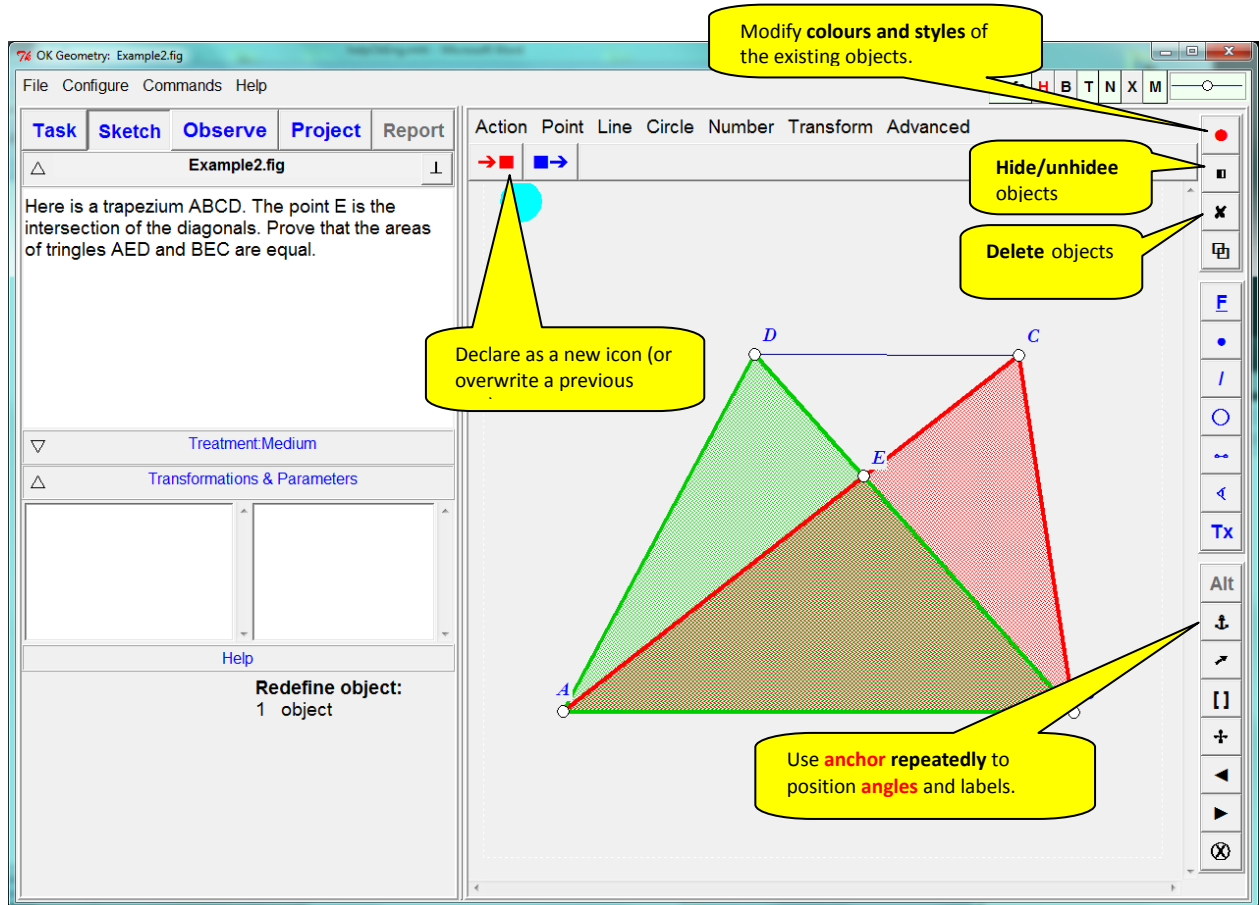


Figure 17

Fill the form that appears (Figure 18) (you can write and edit comments also later). Note that a paragraph in the comment can be declared as: definition/description (D), claim/hypothesis (H), argumentation (A), comment, and formula. Figure 18 shows a commented form for the relation that triangles ABD and ABC have the same area. Click OK to store the relation into an icon.

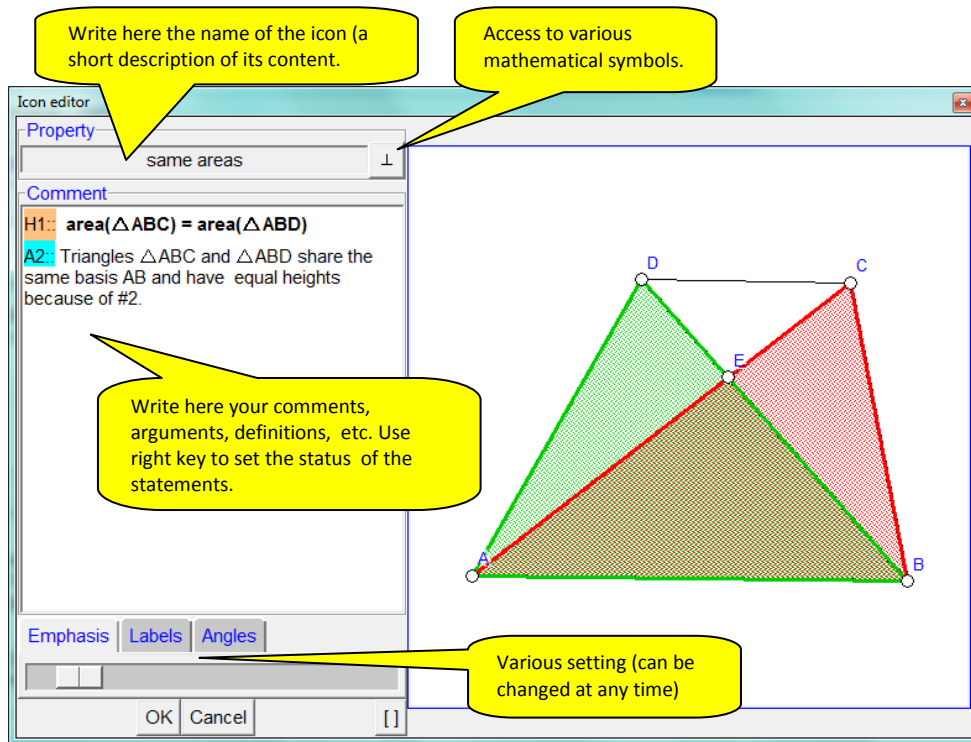


Figure 18

Once you have selected the relevant instances of properties you proceed to organise them in for documenting them or even to build up a proof of a claim. In the main task bar click the **Project** button (Figure 19). On the left pane you see the previously selected instances of properties, while on the right pane there are icons representing the relations. You can move around the icons and change the sequential number of icons. If you want to change the comment to of an icon, simply double click it.

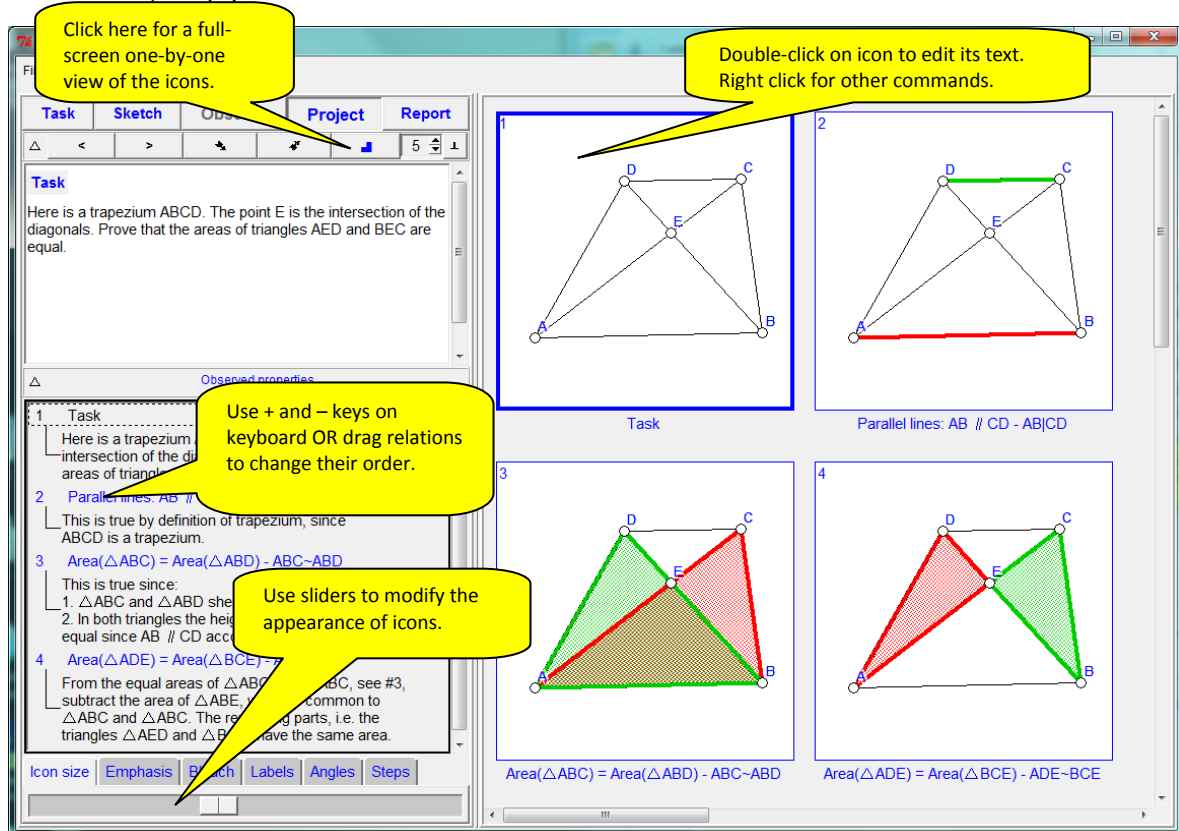



Figure 19

In order to change the sequential number of the emphasised icon either a) use the + and – key on the keyboard or b) right-click on the icon and apply the command *Set icon number*. Other attributes of the icons (size of icons, size of labels, emphasis and bleaching effect, size of angle notation) can be changed with sliders (bottom left of the display).

The icons of a project can be presented all at once (Figure 19) or one-by-one (Figure 20). To present the icons one-by-one click on the  button. Then use the presentation buttons to move between icons. Note: As explained latter, the proof can be organised in a linear fashion or in a tree-like fashion. In the latter case it is possible to set the level of details in the proof and navigate in the steps of the proof.

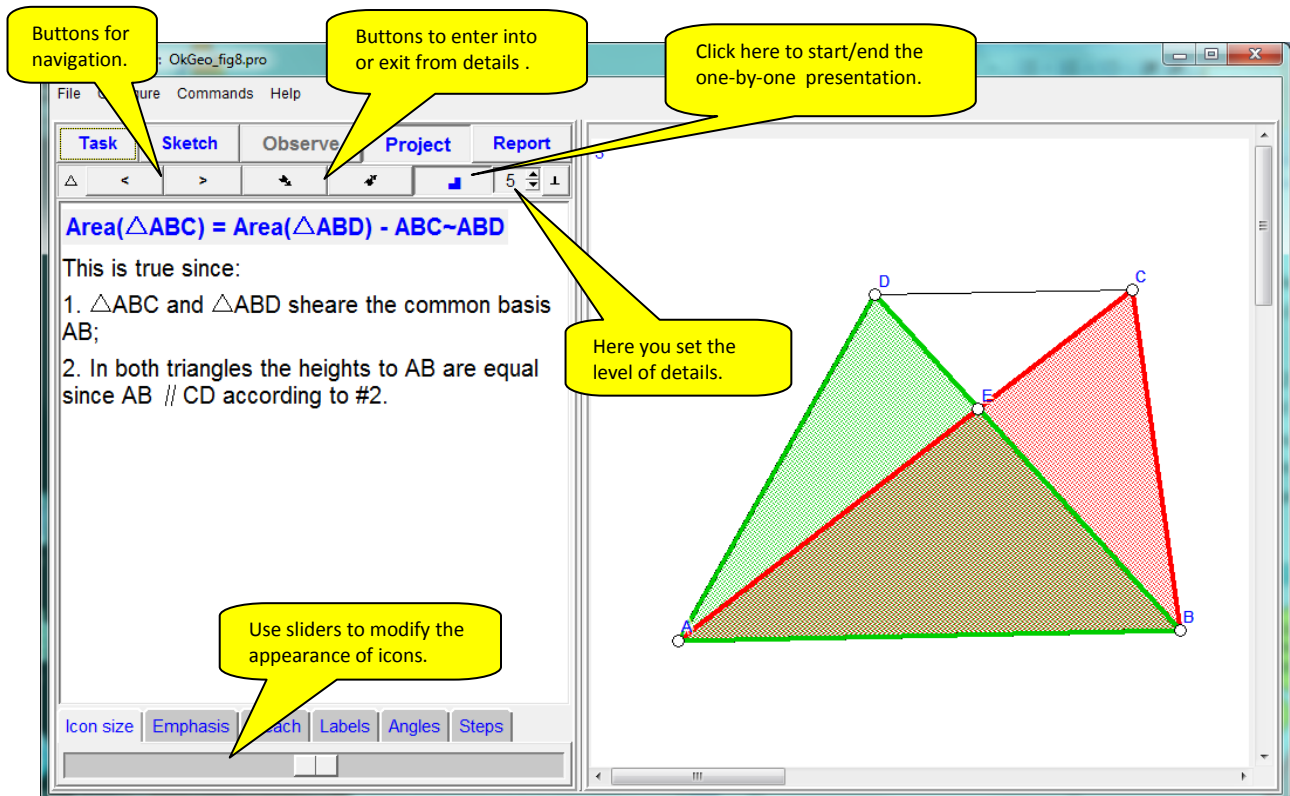


Figure 20

Finally, a report can be made from the selected properties and the added comments. The report can be stored in a .htm, .doc or .tex format. Functionally, the report can be either a memo of the found relations or a classical two column proof of a statement (as built by the user). To make a report click **Report** on the main tool bar and follow the instructions. In the form that appears you are asked for some data (Figure 21) the name of the report file and then to fill a form like the form below:

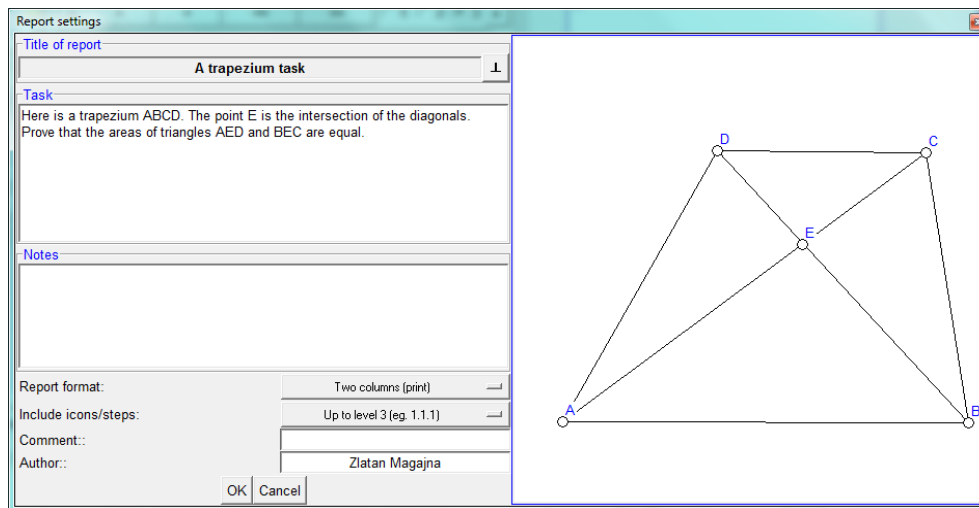
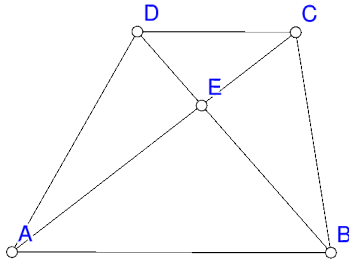


Figure 21

Here is how a report might look like:

A trapezium task

1 Task



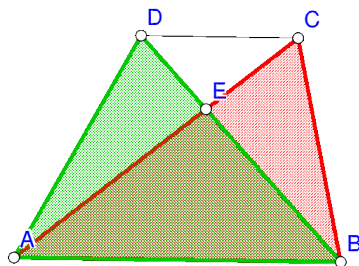
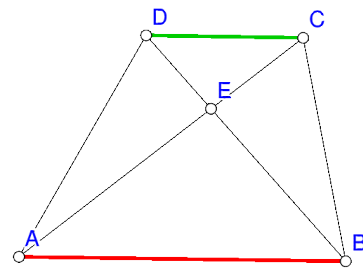
Here is a trapezium ABCD. The point E is the intersection of the diagonals. Prove that the areas of triangles AED and BEC are equal.

Comment:

2 parallel lines

Claim 1 $AB \parallel CD$

Argument 1 By definition of trapezium, since ABCD is a trapezium.



3 same areas

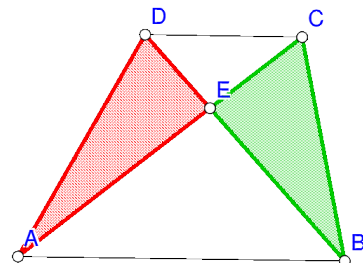
Claim 1 $\text{area}(\triangle ABC) = \text{area}(\triangle ABD)$

Argument 1 Triangles $\triangle ABC$ and $\triangle ABD$ share the same basis AB and have equal heights because of #2.

4 areas of triangles

Claim 1 $\text{area}(\triangle ADE) = \text{area}(\triangle BCE)$



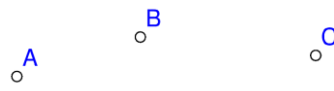
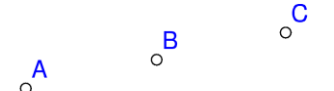
Argument 1 From the equal areas, see #3, of $\triangle ABC$ and $\triangle ABD$ we subtract the common area of $\triangle ABE$. The remaining parts, $\triangle AED$ and $\triangle BCE$, have the same area.



(End of Report)

5 Understanding properties

To begin, consider two similar constructions in, consisting of just three points.

	Construction 1	Construction 2
Construction in a dynamic geometry system e.g. Geogebra	free point A(0,0) free point B(1,0) free point C(2,0)	free point A(0,0) free point C(2,0) constructed point B = midpoint of A and C
Visual presentation in a dynamic geometry system Visual presentation in OK Geometry (exact visualisation)		
Interpretation of the construction in OK Geometry	random point A (several random variants, only one displayed) random point B (several random variants, only one displayed) random point C (several random variants, only one displayed)	random point A (several random variants, only one displayed) random point C (several random variants, only one displayed) B is the midpoint of A and C (in each variant of points A, C, B)
Visual presentation in perturbed visualisation of OK Geometry	 All point often in random position around the original one.	 A and C in random position around the original one, B is always the midpoint of A and C.
Observation of OK Geometry	Nothing is observed.	B is the midpoint of A and C.

Constructions in OK Geometry appear perturbed (unless the visualisation is exact); however, the geometric properties that result from the construction are preserved.

In the analysis of constructions OK Geometry considers

- all **displayed labelled points** (unlabelled points are ignored);
- all displayed lines, circles, and similar objects;
- all lines (displayed or not) passing through any two **displayed labelled points**;
- all circles (displayed or not) passing through any three **displayed labelled points**;
- all conics (displayed or not) passing through any five **displayed labelled points**;
- specific objects with vertices on displayed labelled points (e.g. squares, displayed or not)

The displayed points can be labelled or not, however, **only labelled points are considered in the analysis**. Remember:

- **Point labels** consist of one or more letters, possibly followed by one or more digits and apostrophes. Here are some examples of allowed labels: A, B, b, P, c', D'', E13, t2', Ab. Labels like E13 or Abc are sometimes displayed as E₁₃ and (Abc). For example, the vertices of the triangle $\triangle E4(Ad)P$ are: E4, Ad, P. Labels are case sensitive. Lower case x,y,z cannot be used as labels. Different points cannot share the same label.
- **A reasonable number of labelled points to be considered in a construction is 4-12.** The number of found relations generally rapidly increases with the number of points to be considered in a construction. An excessive number of points to be considered can lead to an unmanageable collection of properties or to an unclear visual representation of the properties. During the analysis it is possible to exclude some labelled points from consideration (see section 8.1 on excluding points from the analysis and 7.5 on changing labels).
- **OK Geometry considers only acute and right angles between lines.** OK Geometry usually observes something about *certain angles or their supplementary angles*. The user has to find out where to consider the angles and where the supplementary angles have to be considered. In Figure 22 (right), for example, with reference to $\angle C$ and $\angle D$ - the ratio 1 : 2 is observed between the two 'angles or their supplementary angles'. Obviously, in this case the supplement of the shown $\angle D$ measures twice as the shown $\angle C$.
- **OK Geometry does not list trivial congruence of angles (angles related to pairs of parallel lines, angles formed by two intersecting lines),** because each pair of parallel lines would produce an immense number of congruent angles. Therefore, when considering the observed properties one should check if parallel lines exist and be aware of the congruence of angles defined by parallel lines and transversal lines. In the trapezium (Figure 22 left), for example, OK Geometry does not list the fact that the angles $\angle DEA$ and $\angle BEC$ or $\angle DCA$ and $\angle CAB$ are congruent.

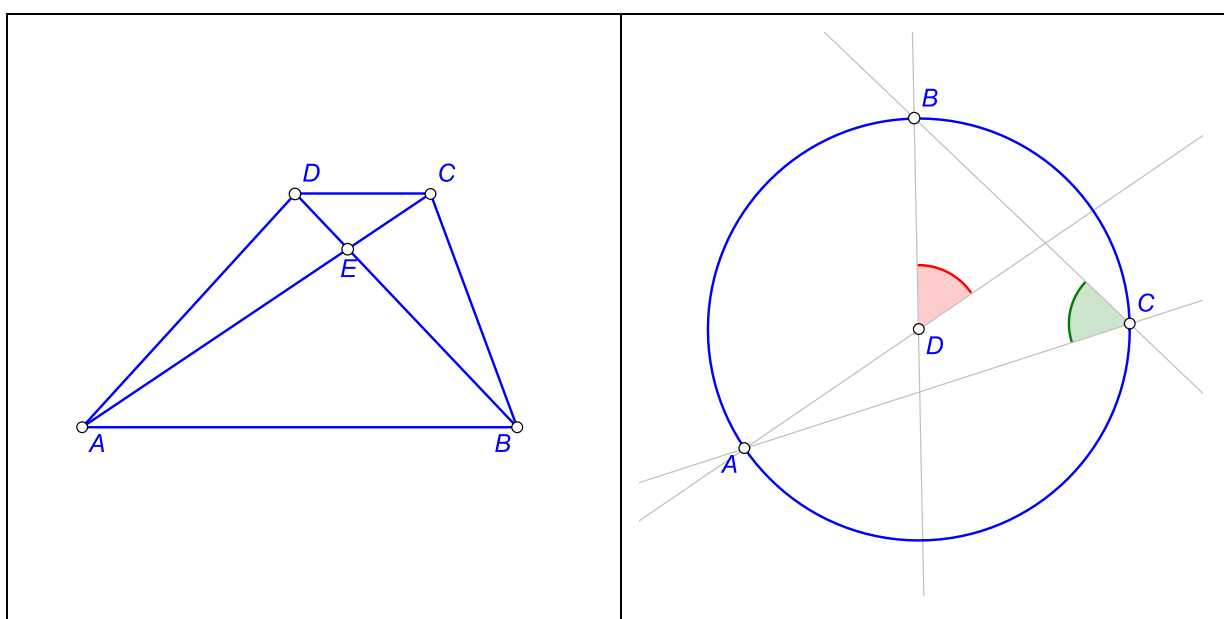


Figure 22

The decision which points to label in a geometric construction is very important. In general, try to label all points that you find potentially relevant to your investigation. In many cases the decision which points to label is rather straightforward, but sometimes a good intuition helps.

6 Working in OK Geometry

When using OK Geometry the workflow consists of three steps. It is always possible to go back to a previous step. The steps are:

Step 1. Obtain a dynamic geometric construction. A construction can imported from a dynamic geometry software or using the Sketch editor, a module of OK geometry. The construction should follow the lines described in Section 5. For more detailed information about importing constructions see Sections 7.1. Detailed instructions for the Sketch editor of OK Geometry are in the Sketch editor manual.

Step 2. Analyse the properties of a dynamic construction. Analysing a dynamic construction is very simple. Just press the button **Observe** in the main menu bar and inspect the properties (some simple adjustments are sometimes needed). See the help manual for Easy mode for a short introduction in this topic. When complex or specific properties are considered, use the advanced operations described in the table below.

Step 3 (optional). Document the observations, eventually organise them into a proof. Any observation can be included into the construction (with the '+' button), can be edited, commented and declared as an icon with the →■ button (see Figure 16). The iconised observations can be later edited (using the **Project** command in the main menu bar) and documented as a report, see the **Report** command in the main menu bar. For more details see Sections **Napaka! Vira sklicevanja ni bilo mogoče najti.** and 11.

OK Geometry is usually used in a simple way: one just creates or imports a geometric construction and uses OK Geometry to obtain a list of properties to be studied. It is also easy to produce a report, if one decides to do so. However, an advanced user probably feels the need to control the process, e.g. to edit a construction or to decide what geometric properties should be checked. For this purpose special operations are available. The table below summarizes common steps and the related specific operations in workflow.

Purpose	Simple operation	Specific operations
Import a construction from Cabri, Cabri Express, GeoGebra, Cinderella, JGEX, Sketchometry or OK Geometry. Edit a construction or make a new one in OK Geometry.	Press the button Task in the Main menu bar and import the construction. Press the Sketch button to edit or create constructions.	Overviewing dynamic constructions made in various dynamic systems Importing constructions from Cabri, Cabri Express, Geometry, GeoGebra, Cinderella, C.a.R. (Compass and Ruler), JGEX, Sketchometry Making a construction in OK Geometry

		Editing geometric constructions Labelling points Writing text related to a task Exporting figures Saving and reading constructions and projects Recovering the work
Obtain a list of properties of the construction.	Press the button Observe in the main menu bar.	Setting the points to be considered in the analysis Setting the level of analysis Taking into account the symmetries of geometric constructions Perturbation radius Detecting properties Interpreting the list of detected properties Visualising a relation
Select and comment relevant properties.	Click the relation to visualise it. Edit, comment and iconise selected relations	Searching specific relations Adding displayed objects to the construction Comment a relation or icon Images and formulae in comments
Organise the selected instances of properties into a document.	Press the Project button to visualise the selected properties. Organise the properties by dragging the icons. Double click them for editing.. Right click the properties or icons for additional operations.	Organising the icons Commenting icons Editing icon's construction Setting a scene Managing scenes Conforming icons Presenting the project Saving the project
Create a report .	Press the Report button and follow the instructions.	Producing reports

Additional themes	Configuring OK Geometry
-------------------	---

7 Geometric constructions

7.1 Importing constructions

OK Geometry imports dynamic constructions from the following software:

Software program	Imported file format (extension)
Cabri Geometry (Cabri Geometry, Cabri Geometry II, Cabri Geometry II Plus),	.fig
Cabri Express,	.clmx
GeoGebra,	.ggb
Cinderella,	.cdy
C.a.R. (Compass and Ruler) or Z.u.L.,	.zir
Sketchometry,	.jc
JGEX (Java Geometry Expert),	.txt
OK Geometry.	.p; .pro

To import a construction click the **Task** button in the main menu bar and select the file you want to import. You can access to this command also from menu *File/Open construction* or from menu *Commands/Task*. OK Geometry imports geometric objects, some visual attributes of the objects and, obviously, the geometric construction for the analysis.

Here is a detailed information on how various objects are imported into OK Geometry:

Points. Try to label points as required by OK Geometry (See Section 5). In any case, the point labels can easily be changed in OK Geometry.

Other objects. OK Geometry imports many types of geometric objects: **points, lines and line segments, vectors, axes, polygons, circles, arcs, loci, geometric measurements, numeric values, calculations, geometric transformations, constructions, and also macros.**

Note that:

Labels of objects other than points are ignored.

Vectors and axes are imported in OK Geometry but are not displayed.

Rays are imported as lines.

Polygons and polylines are imported, but in some file formats mismatches can occur when polygons or polylines are intersected with other objects. The very safe way of intersecting a polygon with another object is to work with separate segments of polygon or polyline.

Properties of objects. Relevant attributes (visibility, fixed objects) are, in general, preserved as well as the colour and visual appearance of objects. Use the command *Configure/General options/Import attributes of objects* to set whether colour and style of object should be (reasonably) preserved when importing objects.

Text. All text entities are accumulated and imported into OK Geometry as the task description.

Magnification. The construction should be made in a reasonable magnification (e.g. between 1:1 to 1:10). An excesses in this respect can lead to observation errors in OK Geometry.

7.2 Over-viewing files from various dynamic systems

The command *File/Inspect files* allows the visual preview of various formats of files in a directory (Figure 23). A previewed construction can be accepted (imported into OK Geometry).

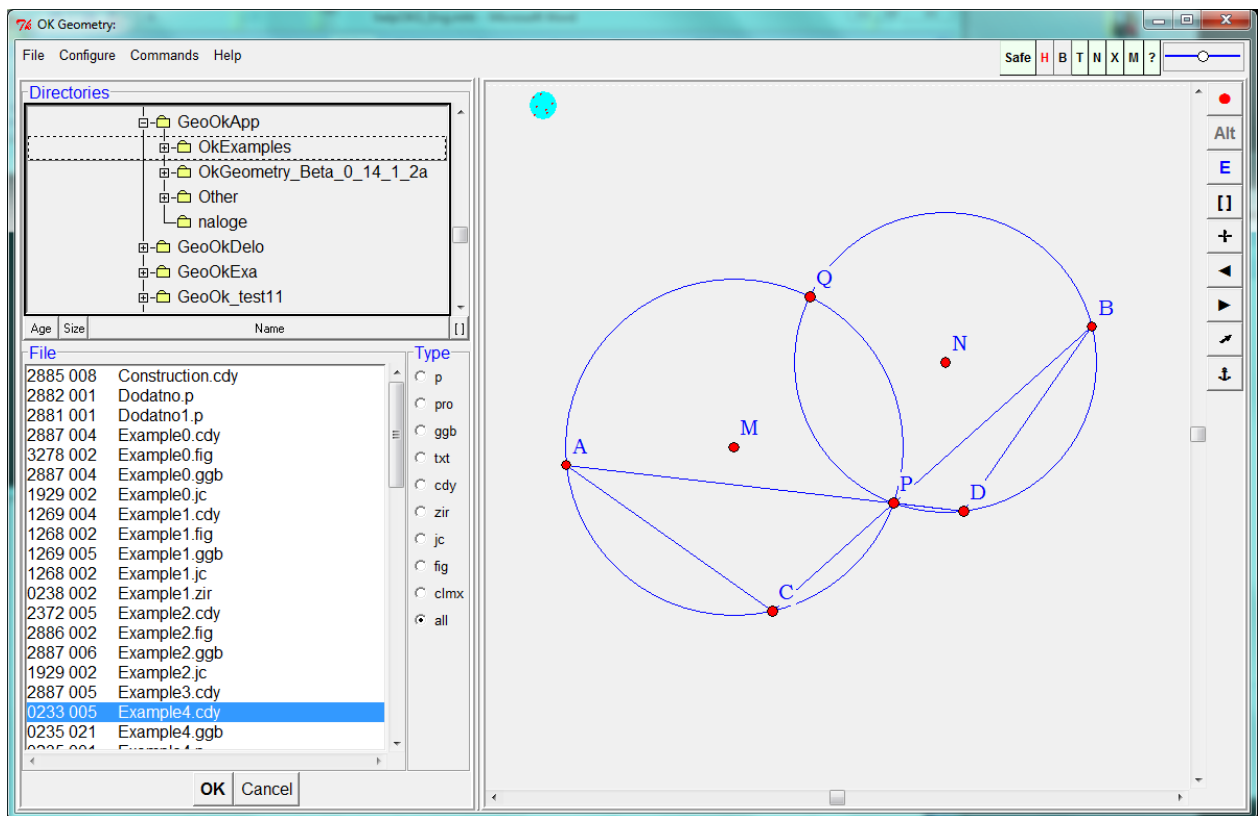


Figure 23

7.3 Creating and editing constructions in OK Geometry

You can edit or make new geometric constructions in the Sketch editor, i.e. in the Sketch mode of OK Geometry. You can access the Sketch editor by pressing the **Sketch** button in the main menu bar or via the menu *Commands/Sketch*. To exit from the Sketch editor press any button in the main menu bar.

The Sketch editor is similar to common editors of dynamic geometry. However, due to specifics of the program, there are some simplifications (e.g. no animation), some specific emphasis, and some specific

commands (e.g. commands for complex and implicit geometric constructions). The Sketch editor of OK Geometry allows to 'construct' a configuration even if one does not know how to construct it as Euclidean construction. OK Geometry then helps to observe the configuration, and perhaps to discover properties one was not aware of and to find ways to do a proper construction and prove its correctness.

Please, refer to OK Sketch Editor Reference for information how to use the editor. There you can also find several examples.

7.4 Exporting figures

The displayed construction can be exported at any time as an image in various formats using the *File/Export image* command in the main menu or the context menu (right click on the construction pane). Note that images of constructions are always exported in the exact visualisation mode. The image can be exported in three ways:

- Via clipboard for fast copy/pasting of image into another application.
- Via a file in a raster format. You can choose among the commonly used formats (PNG, GIF, JPG, BMP, WMF, TIFF). Such formats are compatible with most commonly used applications.
- Via a file vectorised format: (PS (PostScript), SVG, EMF, PDF). These formats give high precision images in applications that can process them.

Note. Exporting to EMF, PDF format as well as exporting symbols in figures requires a prior installation of Inkscape software¹, which is freely available.

Exporting strategy. Here is an efficient way for exporting images of constructions.

- Obtain a desired display of the construction. Note that only the indicated square area in the construction pane is exported. Then call the export command.
- Decide about the desired format/mode of export. If your option is the clipboard or a file in raster format, set the Image quality and image size. In case of a vectorised format choose appropriate size of figure in the document to be created.
- Use the export button (Clipboard, ToFile(img), ToFile(vect)), select the file and execute the export into the document.
- Often the export result are not as expected: The labels are too small or too big, the lines too narrow or too wide, etc.. At this stage correct the Label size, Lines width, and Point size in the export form.
- Repeat step 3 and to obtain the desired result.

Note. Constructions can be also exported as **GeoGebra static files** (and not constructions). Such files can be imported in GeoGebra, eventually modified there, and exported from GeoGebra to other formats.

¹ Inkscape is a free software under GPL license. It is available at <https://inkscape.org/>.

7.5 Labelling points

Labelled points play an essential role in the analysis with OK Geometry since all lines, circles, conics, angles etc. ‘passing through’ the displayed labelled points are considered in the analysis. The labels of points should conform to some simple rules as described in Section 5. The labels of points can be set at the time of making a construction or just before OK Geometry the analysis. All points that are potentially important for the analysis should be labelled.


You can access the commands for labelling points in three ways:

- Use the button for labelling (a button with the current label inside). A letter P (underlined) button asks for a label to be applied. A letter P (not underlined) button labels an unlabelled point as P, and deletes the label if the point is labelled. (See the Sketch Editor Reference for details.)
- Right click anywhere on the visualisation pane. In the contextual menu click the *Labels* sub-command and then choose the appropriate labelling command (see below).
- When in **Sketch** mode – you can also select the appropriate commands in the editor menu (e.g. *Action/Labels* in the leftmost group of commands in the editor menu).

Note: In order to apply a labelling command, first choose the command, then apply the command to (one by one) points you plan to label, re-label, or un-label.

Note: Any change of labels can cause inconsistencies with the found relations in a previous analysis.

Here is a brief description of the commands for labelling points.

Command	Meaning	Comments
Auto label	<p>Let the computer choose the label of a selected point.</p> <p>Clear the label of a labelled point.</p>	<p>The point you click on is labelled with the first available letter after the last applied label.</p> <p>If the selected point is already labelled, its label is cleared.</p> <p>To auto-label more points, just click them one by one in the order you want them to be labelled.</p> <p>The label of the first point can be set with the command <i>Label vertex</i>.</p> <p>Note: A quick way to manually set the next label to be used is to press Alt+S.</p>
Label vertex	<p>Choose the label of a selected point</p> <p>Change the label of a selected point</p>	<p>First click the point you want to label or change the label. Then write the desired label of the point. The program does not accept any invalid label.</p> <p>To manually label more points repeat the operation for each point or skip to <i>Auto label</i>.</p>
Label all	Label all unlabelled displayed points	All shown unlabelled points are labelled in the order as they were defined. Existing labels of points are not changed.
Clear label	Delete the label of selected points	<p>The command deletes the label of the point you click on.</p> <p>To delete the labels of more points just click on each of these points.</p>
Clear all labels	Delete all labels	The command deletes the labels of all points. After this operation all points in the construction are unlabelled.
Anchor	Position the label	The positions of the labels can be modified using the Anchor command or the  button (afterwards try also the mouse scroll button). Labels can also be dragged around the referred point.

All points that are potentially important for the analysis should be labelled. Points that are clearly irrelevant should not be labelled: this can be achieved by hiding the points or clearing the labels.

7.6 The text of a task

A geometric construction is usually accompanied by a text describing the construction or the task associated with the construction. Such a text can be written in the dynamic construction (if imported) as a common text. It can be also written or edited at any time in the “Task” section of the information

pane. The text can consist of the common text, but it is also possible – in a certain way – to include symbols, mathematical formulae, and pictorial information.

Inserting symbols in text

To insert mathematical or other symbols in the text use the symbols table available with the \perp button on the title line (Figure 24).

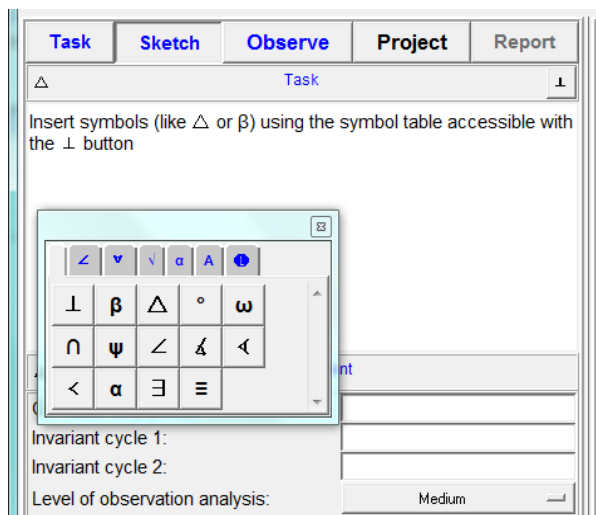


Figure 24

Inserting formulae and pictorial information in text

Images and formulae can be copy-pasted in the text with a right click on the text area (*Image/Paste image* command). To obtain the desired size of the picture in text set the paste size accordingly.

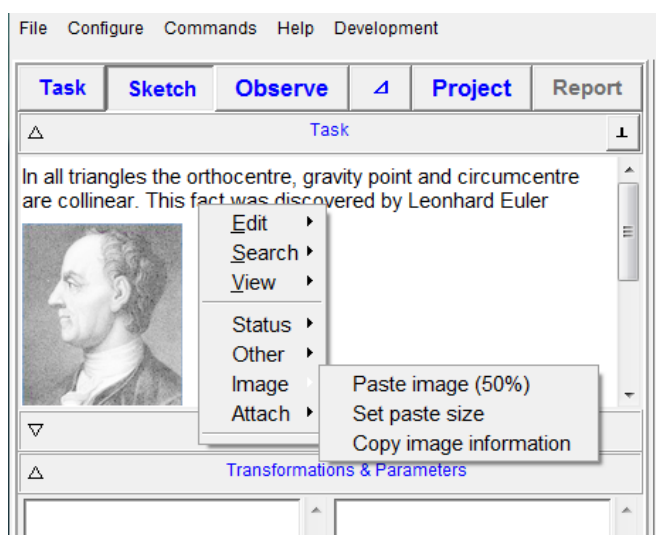


Figure 25

Mathematical formulae can be imported using an appropriate editor. Apart from the pictorial format of the formula, which is imported as image, it is possible to import also the source of the formula. The source has no function in OK Geometry, except that can be retrieved in case one would like to make changes (in the appropriate editor).

Inserting links

Links to files and web pages can be inserted with the Attach command in the context menu (right click on the text area).

7.7 Inspecting the construction steps

To visualise the sequence of construction steps use the command *Commands/Construction steps*. The command can also display the scenes of a construction.

7.8 Saving and retrieving constructions and projects

OK Geometry stores data in two types of files. The current construction is saved in file with the **.p** extension, the current project is saved in file with **.pro** extension.

Saving constructions. To save the current construction to a file, use the *File/Save construction* command from the menu. The construction is saved in OK Geometry format. Note that the saved construction also contains the information about the Task section and Treatment section of the information pane but does not contain any information about icons and properties found or considered.

Retrieving constructions. A saved construction can be retrieved with the *File/Open construction* command from the menu or with the Task button.

Saving and retrieving projects. Current projects with commented icons comments and underlying dynamic constructions are saved and retrieved in files with **.pro** extension. To save the current project use *File/Save project* command from the menu. The work that was saved in a **.pro** file can be retrieved using the command *File/Open project* from the menu.

7.9 Recovering the current work

Recovering the current work. OK geometry continuously saves the current work. The last saved work can be recovered using the *Configure/Recover last session* from the menu command.

8 Analysing constructions

In most cases there is no need to configure the analysis. Simply press the **Observe** button in the main menu bar. Occasionally, however, simple adjustments might be helpful.

8.1 Setting the points to be considered in the analysis

Why setting the points for the analysis?

OK Geometry in a construction only takes into account the labelled points (see Section 5). It is therefore important that all potentially relevant points are labelled. However, an excessive number of labelled points creates several problems: The list of relations, observed by OK Geometry, can be so extensive that it is difficult to identify the interesting ones among them. Also, the observation process can take a

while, and the detection of certain properties can be unreliable. Thus, it is advisable to restrict the analysis to 4-12 points and, eventually, repeat the analysis on various sets of points of the same configuration. For this reason OK Geometry allows to specify which points to consider in the analysis without changing the construction itself.

The points to be considered in the analysis are declared in the **Treatment section** of the information pane. The points to be considered in the analysis should be entered in the entry **Consider only points**. If the entry is empty, all labelled points are considered in the analysis, otherwise only the declared points are considered. If there is an exclamation point at the beginning, the indicated points are not included in the analysis.

Figure 26

Here are some examples for the entry **Consider only points**:

empty – All labelled points in the construction are considered in the analysis. This is the default assumption.

AB'C1(DG)R – Only points A, B', C₁, DG, and R are considered.

!ADH – All labelled points *except* A, D, and H are considered.

The considered points are saved as a part of the geometric construction. When you retrieve a construction (in OK Geometry format) you retrieve also the information which point are considered in the analysis.

8.2 Symmetries in geometric constructions

Using this advanced feature we put an additionally structure in the set of detected relations, so that they are much more reliable and also easier to manage. We can apply this feature if we are looking for properties that are invariant under given rearrangements of vertices. It is possible to choose between two types of rearrangements:

Cyclic rearrangements. We use the notation like ABC DEF to call for properties that are preserved if the vertices A,B,C, and D,E,F are changed cyclically to C,A,B, F,D,E or to B,C,A, E,F,D. In other words, we require that the observed properties are invariant under any cyclic permutations of ABC and the respective cyclic permutations of D,E,F.

Complete rearrangements. We use the notation like $*ABC \text{ DEF}$ to call for properties that are preserved if the vertices A,B,C, D,E,F are rearranged to C,A,B, F,D,E or to B,C,A, E,F,D or to B,A,C, E,D,F or to C,B,A, F,E,D or to A,C,B, E,D,F. In other words, we require that the observed properties are invariant under permutations of ABC and the respective permutations of D,E,F.

Let us explain this in with some simple examples.

Given is a triangle ABC. The inscribed circle (with centre G) of this triangle touches the sides of the triangle in points D, E, and F (see Figure 27).

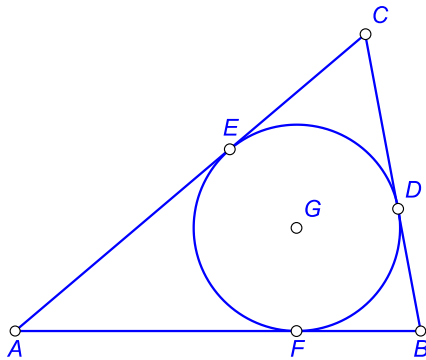


Figure 27

In this configuration there are, for example, 6 right triangles:

AEG, BDG, CEG,
AED, CEG, BEG.

You can easily see that these triangles refer to, basically, the same property. We can reduce the 6 instances of this property to just one, for example AEG. All the other instances are obtained by permutations of the vertices A,B,C and respective permutations of D,E,F. We symbolically describe this invariance by writing $*ABC \text{ DEF}$ in the entry **Invariant cycle 1** (Figure 28). OK Geometry lists 6 right triangles. Note that 5 instances of the right triangles are written in blue colour, indicating that these instances are, basically, 'repetitions' of some other instance of property. The blue coloured instances of properties are thus obtained from other instances properties by applying the permutations in the Invariant cycle entry or entries.

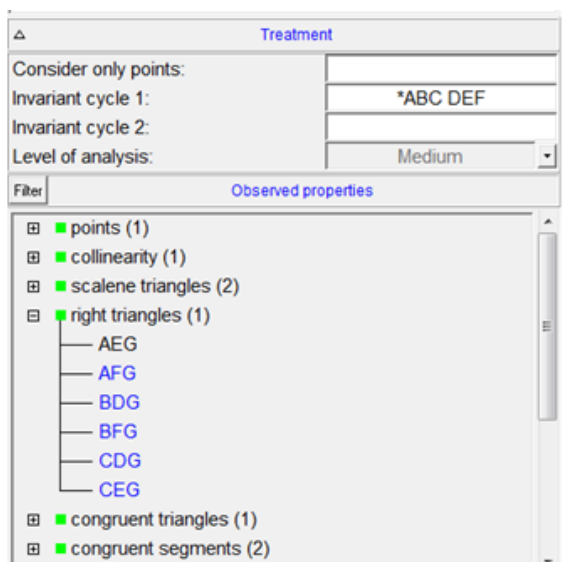


Figure 28

Consider now a more sophisticated example: a kite (Figure 29). The kite has a line of symmetry, thus its properties are preserved if B and D are interchanged. Setting the entry **Invariant cycle 1** to BD would therefore reduce the number of relations. Note that the geometric properties are preserved also if A and C are interchanged independently of B and D. Thus setting the entry **Invariant cycle 2** to AC would still reduce the number of relations.

What happens if the **Invariant cycle 1** is set to ABCD and the **Invariant cycle 2** is left empty? The property that the triangle ABD is isosceles is not listed since the triangle BCA (obtained by a cyclic permutation of the vertices) is not isosceles. However, the fact that the line AC is perpendicular to BD is listed it is clearly invariant under the cyclic permutations of vertices A,B,C,D.

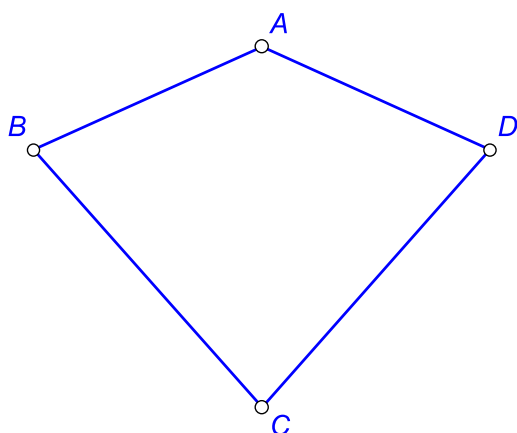


Figure 29

Some additional notes:

- By default the **Invariant cycles** entries are empty, thus OK Geometry does not assume that properties need to be invariant under any permutation.
- It is perfectly ok if only one invariant cycle is considered.
- The syntax of each entry is evident from the examples above. Note that each entry should consist of one or more cycles of the same length. A given point can be present in each entry at most once.
- The invariant cycles are considered as a part of the construction. Thus they are saved together with geometric constructions. When you retrieve a construction (in OK Geometry format .p) you also retrieve its invariant cycles.

8.3 Setting the level of analysis

You can choose between these possibilities: Simple, Medium, Advanced, and Complete. The default level of expertise is set in the menu command *Configure/General options/Observe* under the entry **Default level of analysis**.

The level of analysis for the current task is set in the Treatment section of the Information pane, as shown in Figure 30.

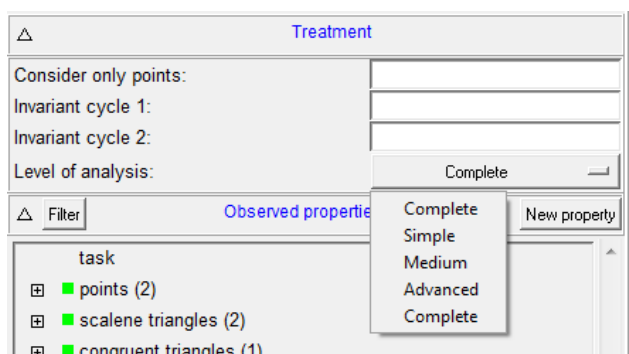


Figure 30

Here is a very short description of the levels of expertise:

Level	Description of the considered properties
Simple	Basic properties of geometric objects, specific shapes (of triangles), congruence
Medium	Simple + ratio of lengths and angles, similarity
Advanced	Medium + advanced ratios of lengths, points on conics, golden ratio, cross-ratio
Complete	Advanced + relations that often give rise to a big number of properties (e.g. pairs of angles with the same sum) . This level is usually avoided.

Detailed information about which properties are considered at a particular level is available via the menu command *Configure/Considered properties*.

Choosing the level of expertise is, obviously, a matter of user's expertise in geometry and of the content of the task. In general, choose among Simple, Medium, and Advanced level. The Complete level should be used with care, since the some properties, specific to this level, sometimes generate a long and unmanageable list of observed relations.

Note. OK Geometry Plus observes many additional properties related to triangle geometry.

9 Detecting properties

9.1 The list of found properties

The main feature of OK Geometry is the ability to detect geometric properties in a geometric construction. This is achieved with the **Observe** button in the main menu bar or with the *Commands/Observe* command in the main menu. The command **Observe** explores whether the objects (determined by the labelled points) in the current geometric construction have certain geometric properties.

The **Observe** command produces a list of properties and instances of properties of the studied geometric configuration. The list is rather self-explanatory, but some used conventions should be mentioned. Figure 31 shows an example of a list produced with the **Observe** command. For each considered property there are 4 possible outcomes:

No instance of a property was found. In this case the property is not listed. In Figure 31, for example, no right triangles were found, since the property is not listed.

Some instances of a property were found. In this case the property is listed with an expansion sign (a + in a small square) in front of the property and the number of instances after the property. In Figure 31, for example, OK Geometry has found 4 instances of equilateral triangles. To see the found instances, just click the expansion sign.

The property was not checked. In this case the property is listed without the expansion sign and followed by a question mark. In the considered example (Figure 31) this is the case for the equality of ratios of areas of triangles. There are two reasons why OK Geometry does not check a property. There can be so many instances of properties to check that the computation would take too much time. It is also possible that OK Geometry finds that the observation of the property is too unreliable. There are at least two ways to overcome such a situation:

- The preferred way is to reduce the number of involved vertices (see 8.1).
- You can increase the limit for the allowed number of instances of properties to be checked (see Section 12.2). Then repeat the **Observe** command.

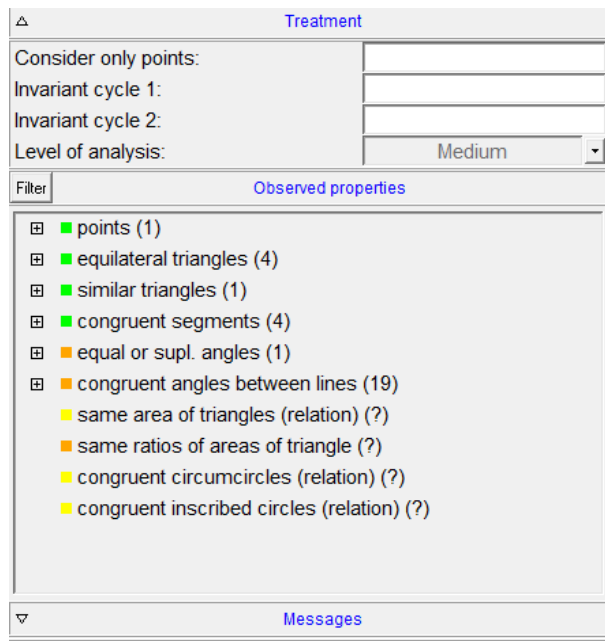


Figure 31

Notice the coloured squares in front of the names of properties. The colour of the square indicates the probability of error in observation:

green square - the probability is extremely low;

yellow square - the probability low;

orange square - the observed relation is possibly a result of chance;

red square - OK Geometry has detected a suspicious feature in the construction that can jeopardise the validity of the observation.

The reliability of the results, obtained with OK Geometry, **is significantly improved**

- if symmetries in construction are considered (see Section 8.2) or
- if the number of considered points is reduced or
- if the level of perturbation is increased (see menu command *Configure/General options/Perturbation magnitude*)

On the other hand, the observations are reliable

- if the construction is based on optimisation or the construction is implicit or
- if the number of considered points is big.

9.2 Instances of properties

If there is an expansion mark in front of a property, then the instances of the property can be displayed by clicking the expansion mark. The interpretation of the listed instances of properties is often straightforward, especially if there is its graphic representation in the visualisation pane. In the situation presented in Figure 32, for example, OK Geometry has found three instances of pairs of congruent triangles.

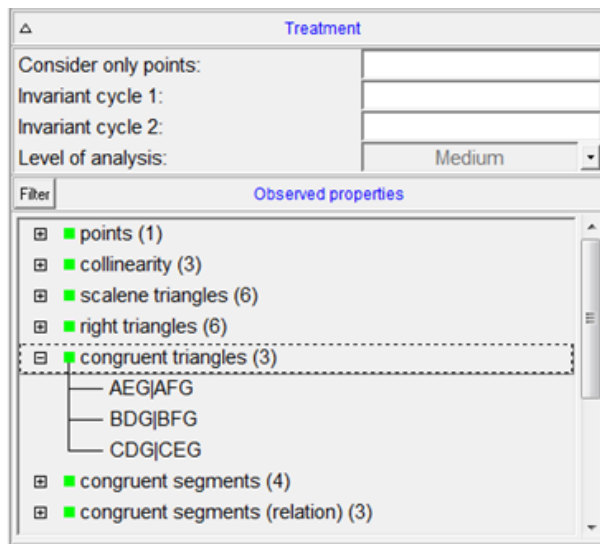


Figure 32

Usually, there is no need to bother about details in symbolic expressions of relations. Nevertheless, here are the basic rules:

- Objects are represented by juxtaposition of points that define them. From the name of the property we infer the type of the involved objects (e.g. collinearity is clearly about points on lines, similar triangles is clearly about triangles). The objects are then identified by points that define the objects. For example:
- Lines are denoted by two or more points on a line (e.g. AC, AGTU). Note that the order of points **does not** indicate the order of the points on the line.
- Circles are denoted by 3 or more points on the circle (e.g. ABC, ABMR). Note that the order of vertices **does not** indicate the position of the points on the circle.
- Triangles are denoted by the three vertices (e.g. ABC, AMP). The order of vertices has **no** geometric meaning.
- Angles between lines are shown as AB#CD (in this case the angle between the line AB and the line BC; here the vertex of the angle needs not be a part of the construction). Similarly ABCD#EF represents the angle between the line containing the collinear points A,B,C,D and the line EF. Also in this case **we do not distinguish between an angle and its supplement**. More precisely: ABCD#EF denotes the un-oriented angle (between 0 and 90 degrees) between lines ABCD and EF.
- Each property is about objects of the same type. Usually the presented objects have the same property. However they are listed using separation marks of different hierarchical order. In practice you need not bother of the meaning of the separations symbols – they are self-evident, especially when we visualise the relation. The meaning of the separation marks (in order from high precedence to low precedence) is:

	parallel lines, congruent segments, congruent triangles, etc.
#	angle sign
~	same quantity (e.g. angle, area, perimeter), similar triangles
+	sum of quantities (lengths, angles, etc)
:	ratio of quantities, also tangency

- , enumeration of objects
- = same numerical values (e.g. same sum, same ratio, etc.)
- ; separator of enumerations

Here are some examples:

Similar triangles: ABC APD~EFG	In this case the triangles ABC, APD, and EFG are similar, but ABC and APD are even congruent. Note that the order of vertices means nothing, i.e. ABC APD does not mean that AB is congruent to AP.
Sum of lengths: AP AC+EF=CD+CE BC.	In this case the segments AP and AC are congruent, and so are the segments CE and BC. Furthermore the lengths AP+EF, AC+EF, CD+CE, CD+BC add to the same sum.
Same angles: AB CD#EF GH [=30 deg].	In this case OK Geometry has found an angle of 30 degrees. Actually there are two pairs of parallel lines AB is parallel to CD, and EF is parallel to GH. The angle between the directions of these lines is 30 degrees.

9.3 Visualising relations

To visualise a relation, just click on the displayed instance of property in the Information pane (see Figure 16). For simple properties an iconic representation with obvious interpretation appears in the visualisation pane. For complex properties (e.g. when a property involves many objects) you need to visualise smaller groups of involved objects or even single objects. There are two easy ways to achieve this:

- When a property is visualised, note that the property is written also in the visualisation pane, just above the drawn construction. Click with the mouse on an object in the written representation, and the object becomes highlighted. If you click again on the same object then the related groups of object becomes highlighted (see Figure 16).
- A more flexible way of visualising complex relation is via direction keys on the keyboard. With the keys Up and Down you select whether to represent a broader or narrower group of related objects. With keys Left and right you can move between groups of objects. Try also to combine the left and right direction keys with the Shift key (see Figure 16).

9.4 Query objects

The list of all found properties is sometimes very long (e.g. several hundreds of them). An experienced (advanced) user focuses only on certain properties and does not loose time with others. Sometimes one wants to focus only on relations that involve a specific object, for example: *“I feel that this triangle ABK is important. I wonder if it is congruent, similar or otherwise related to some other triangle.”* Or: *“I’d like*

to know how to construct a point if only some of the objects in the configuration were known.” You can find such information easily using the query commands.

Objects in a configuration can be queried when the observed properties are listed on the left pane. The query buttons are positioned on the task bar on the right (Figure 33, right). The query buttons from up to down are:

Command	Meaning
Mark/unmark non-given objects	This command is used to mark objects that are considered as non-given. Object marked as non-given are coloured in pale-green. Such objects are not considered in the query, unless they are the object of query. A typical use of this function is the study of construction problems. In a complete construction we mark objects that need to be constructed from given objects.
Query labelled point	Click on the point to be inspected. On the left pane only properties related to the inspected point are listed.
Query-Segment	Click on the segment to be inspected or its endpoints. On the left pane only properties related to the inspected segment are listed.
Query-Line	Click on the line to be inspected or on two labelled points on it. On the left pane only properties related to the inspected line are listed.
Query-Circle	Click on the circle to be inspected or on three labelled points on it. On the left pane only properties related to the inspected circle are listed.
Query-Triangle	Click on the labelled triangle vertices. On the left pane only properties related to the inspected triangle are listed.
Query-Angles	Click on the labelled points (ray, vertex, ray) or on the rays of the angle to be inspected. On the left pane only properties related to the inspected angle are listed.

When an object is queried, a row of dotted buttons appear above the list of properties (Figure 33, left). The number of dots on buttons indicates the strictness of the query: the less dots, the less extensive information is displayed, i.e. less relations with less objects are displayed.



Figure 33

An example

Given is a line segment AB and a point C on it. Consider three circles with diameters AB , AC and CB with respective centres with respective centres F , D , E (Figure 34). The upper half of the obtained shape is often called arbelos. We would like to inscribe a circle in this figure, as shown in Figure 34.

OK Geometry Sketch editor allows to position a circle that touches three given circles, the construction is in the Examples directory (Example5.p). However, we are looking for a Euclidean construction. For the construction it suffices, for example, to obtain the three touch-points H , I , J (there are other options, of course).

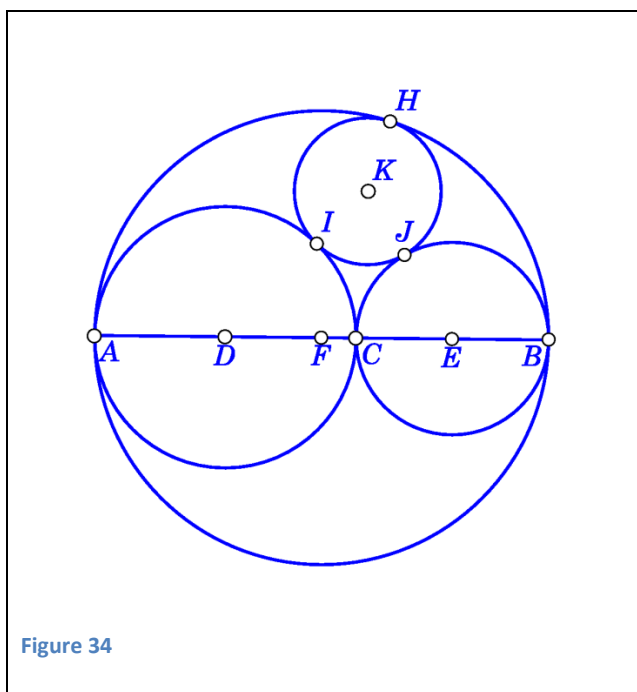


Figure 34

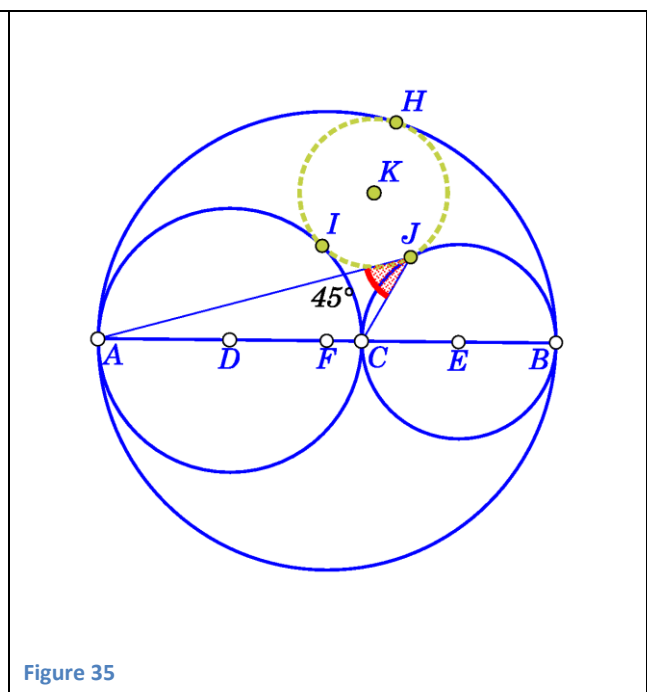
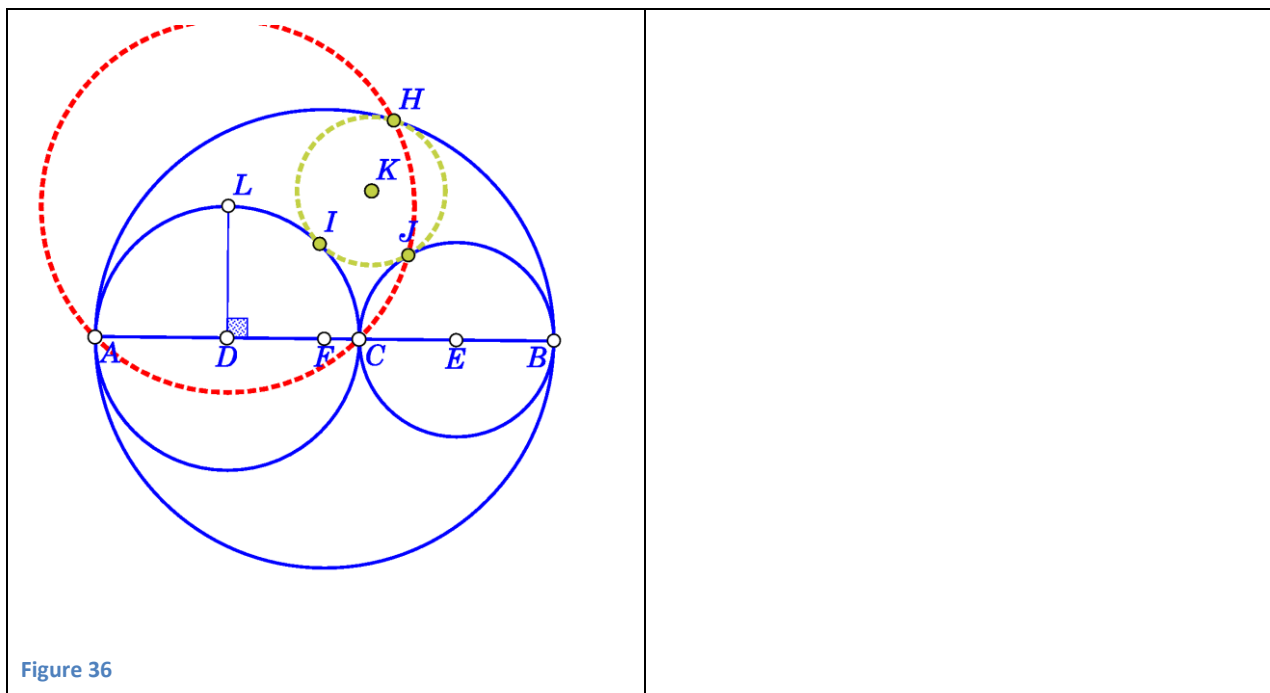


Figure 35



1. First use the **Observe** command to obtain the properties of the configuration in Figure 37 that includes the points H, I, J, K, and the inscribed circle.
2. **Mark** the points H, I, J, K, and the inscribed circle **as non-given objects**. since we do not yet have an idea how to construct them from given objects. The non-given objects become pale-green coloured.
3. Then query a non-given point, for example the point J, with the command **query point**. We obtain a restricted list of properties that involve the point J, but not H, I, K and the inscribed circle.
Some of the properties are related to the fact that J lays on the circle with centre E. However, some properties give a clue (not proof!) to the construction task. For example, the property 'ratio of distances' $AC:AB=CJ:BJ$ allows the construction of J using the Apollonius circle in triangle BCJ.
Perhaps the nicest property is related to angles: The angle $\angle AJC$ measures 45° . This observation leads to hypothesis that J is the intersection of two circles (Figure 36), one of them has L as centre and passes through A.
4. In a similar fashion we hypothesise how to construct points I, J, K. Note that once we have an idea how to obtain J, we can consider J as a given object, etc. Needles to say that the observed properties are mere hypotheses and should be proved.

Note.

- Try different query levels. In construction problems the best level are the more restricted (2 or 1 dot).
- In construction tasks pay attention to 'multiple intersections' properties. Sometimes it is possible to obtain from these properties unusual and unexpected ideas for construction.

9.5 Adding new displayed objects to the construction

Sometimes we want to include the displayed relation into the considered construction. For example, OK Geometry detects two congruent triangle (Figure 37). To include these two triangles into the construction, we can proceed in two ways:

- If we press the red **+** button above the visualisation pane (see Figure 16), the added objects are displayed as emphasized objects.
- If we select the **+** command, the added objects are displayed as ordinary objects (Figure 37).

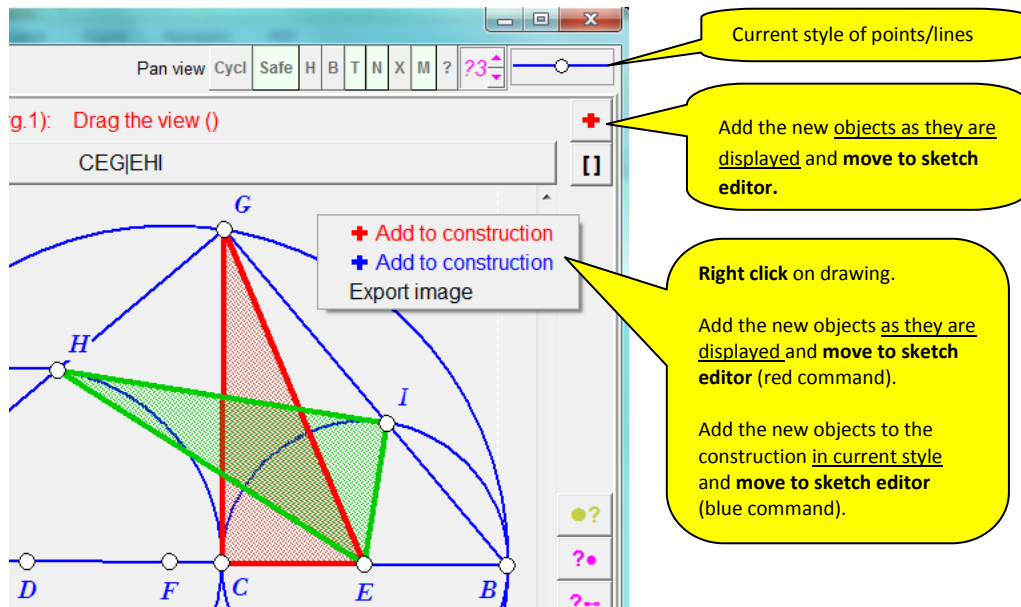


Figure 37

The adding of objects to construction in one or another way brings us to the Sketch editor. In the Sketch editor it is sometimes necessary to make some adjustments to the added and emphasized objects:





- Use the **Anchor** button  (possibly together with the mouse wheel) to *position/select an angle between lines* and to move around the point labels.
- Use the **Colour/Style** button  to change the appearance of objects.
- Use the **Hide** button  or **Delete** button  to hide or delete objects.

Figure 38 shows an example of adding object from a displayed relation. The detected relation (left) was added with the **+** button to the construction (center). The representation was then modified (note the position of angles and labels) with simple editing commands Anchor, Delete, Change colour/style..

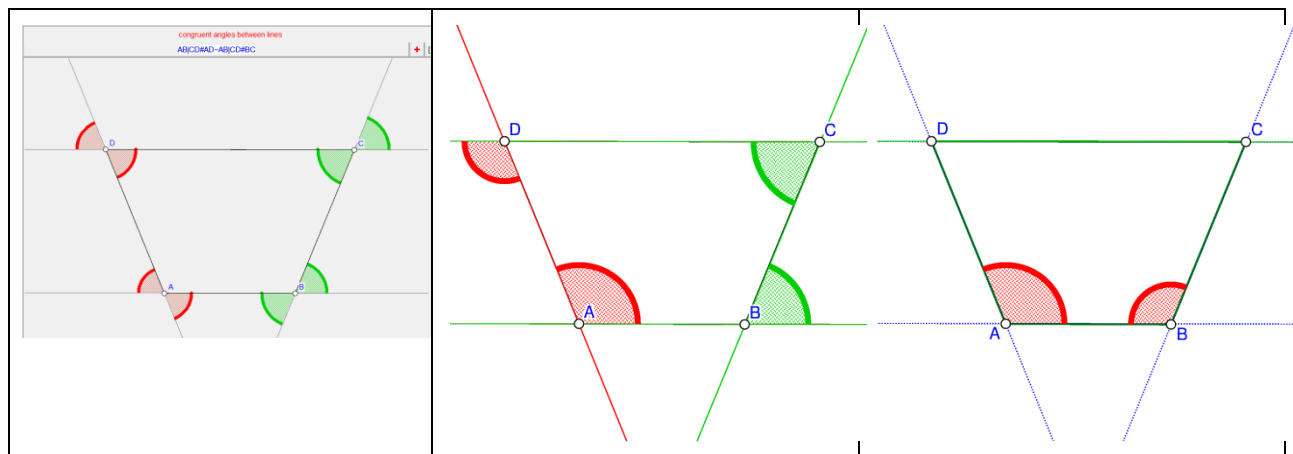




Figure 38

9.6 Declaring a construction as an icon

A construction can be included as an icon into the current project. The first icon of the project is called 'Task' by default. In order to iconise a construction:

1. Declare the current construction as an icon. This is done by pressing the button  above the visualisation panel. Declare the current construction as a **New** icon or overwrite an existing icon.
2. At this stage (or later) it is also possible to comment the icon. See Section 4.2 (Figure 15, Figure 18) for an illustration of this process.

9.7 Retrieving construction from aicons

A project consists of several icons. Each icon contains a construction, which can be retrieved with the  button above the visualisation pane.

10 Menaging the project

A project consists of several icons. Icons can be selected and included into a project for various purposes. Here are some ideas:

- icons show relations in a configuration; by dragging and commenting icons one organises the relations into a deductive structure (a proof);
- icons serve as a memo, e.g. a list of interesting properties of a studied construction;
- icons consists of tasks for the students.

To visualise all icons of the current project, press the **Project** button in the main menu bar. On the left side you can see the list of icon titles, on the right side you can see the graphical representations of the icons, arranged in some way (Figure 39). These graphical representations can be dragged around.

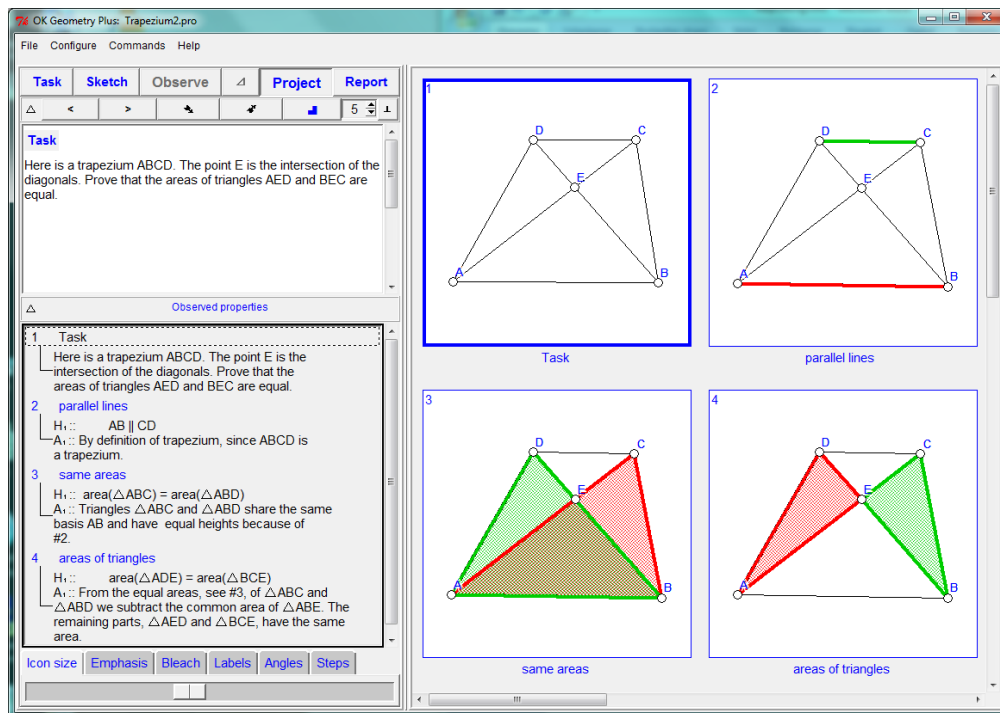


Figure 39

Each icon consists of :


- **The underlying construction.** The underlying construction of the icon can be retrieved using the context menu (right click on the icon and select the command **Edit dynamic construction**). In this way it is possible to work simultaneously with several constructions.
- **The enumeration.** If the icons are organised in a linear structure, they are numbered with the numbers 1,2,3,... In this case, all icons belong to level one. Icons can be demoted to a lower level or promoted to a higher level. A demoted icon becomes a 'sub-icon' of the previous icon of the higher level. For example, icon no. 2 can have sub-icons 2.1, 2.2, 2.3. The Icon 2.1 can contain sub-icons 2.1.1, 2.1.2, etc. In this way the icons can be organised in a tree-like structure.
- **Other information.** Each icon has a title and an optional textual comment.

10.1 Organising and commenting icons

Icons can be dragged around. Other operations on icons are accessible via the context menu (right click):

Command in the context menu	Description
Edit comment	<p>Double click the active icon. In the form that appears you can modify the comment as well as the name of the icon.</p> <p>If the comment is a deductive argumentation, it is possible to specify the status of paragraphs (with a right click on paragraphs). A paragraph can be given the status of a</p> <ul style="list-style-type: none"> • definition/assumption/declaration • claim/hypothesis

	<ul style="list-style-type: none"> • argument/reason of truth <p>Furthermore, a paragraph can be formatted as a comment or a formula.</p> <p>See Figure 44 for an example of such comment.</p> <p>Simple mathematical expressions can be written with symbols available with \perp button. Complex formulae and images in general can be written in other software and copy/pasted to OK Geometry – use Image Paste image command in context menu (obtained with a right click).</p>
Edit dynamic construction	This command sets the icon's underlying construction as the current construction in the Sketch editor. The edited construction can be saved as a new icon or as an existing icon.
Set icon number	<p>In the form that appears set the new position of the active icon. The new position can be on the same level (<i>after</i> an icon) or on the lower level (<i>right after</i> an icon). For example, assume the current icons are 1, 2, 2.1, 2.2. The position <i>after</i> 2 is 3, while the position <i>right after</i> 2 is 2.3.</p> <p>Note that the icon number 1 (Task) cannot be renumbered.</p> <p>You can increase/decrease the number of the active icon on top level using the + and – key.</p>
Promote or demote icon	<p>Icons at detail-level 1 are numbered as: 1, 2, 3 ... Those at detail-level 2 are numbered as 1.1 or 2.4, and those at level 3 like 2.1.4.</p> <p>Demoting an icon increases its level, promoting decreases the level. You can promote (demote) the active icon also by pressing F3 (F4).</p>
Arrange icons	<p>Icons can be arranged on the pane in various ways:</p> <ul style="list-style-type: none"> - in the form of an Array, - in the form of a Cascade, - in a Free form, as they are positioned by the user, - in the form of a Flowchart. <p>The default form is the Array.</p>
Copy / paste icons	In the form that appears just select the icons to be copied. Icons can be pasted to the same project or to another project.
Delete icons ...	In the form that appears just select the icons to be deleted.
(Modify the appearance of the icons)	Use the sliders at the bottom of the left pane to modify the size of all icons, labels or angle notation, visual emphasis of objects, and the bleaching effect.
(Set the displayed detail-level)	The displayed detail-level is set on the navigation bar just below the main menu bar (see Figure 20).

	If the displayed detail-level is set to 2, only icons of level 1 and 2 are be displayed.
Expand/shrink icon	<p>You can switch between showing the whole collection of icons or just one by one icon. To switch between the two modes you can also use the  button below the main menu.</p> <p>One-by-one mode is intended for presentations (see 10.3).</p>
Conform icons	<p>Transfer selected properties of an icon to selected icons.</p> <p>This command adjusts the size of the represented word and/or the represented objects of the selected icons so that they conform with those of a given icon.</p>

10.2 Emphasised, bleached and transparent objects. Auxiliary objects

Objects in icons can be emphasised, bleached or made transparent in Sketch editor. The levels of these attributes can be set during visualisation of the icons with the sliders on the bottom of the left pane (see Figure 19).

Emphasised objects are displayed with variable width. Emphasized object can be dynamically set as more or less prominent during the visualisation of icons, as shown in the case of circle and triangle on Figure 40.

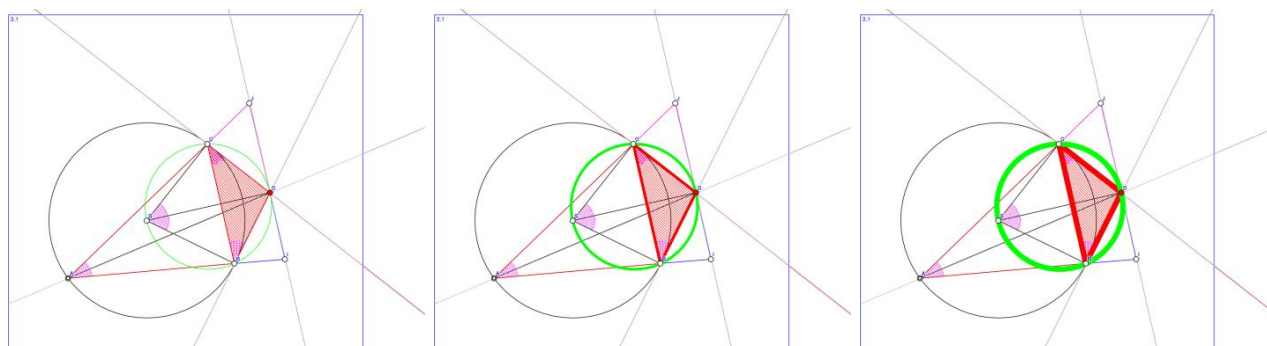


Figure 40

Bleached objects are displayed with variable level of fading. Bleached object can be dynamically set as more or less faded during the visualisation of icons, as shown in some angles and triangles on Figure 41.

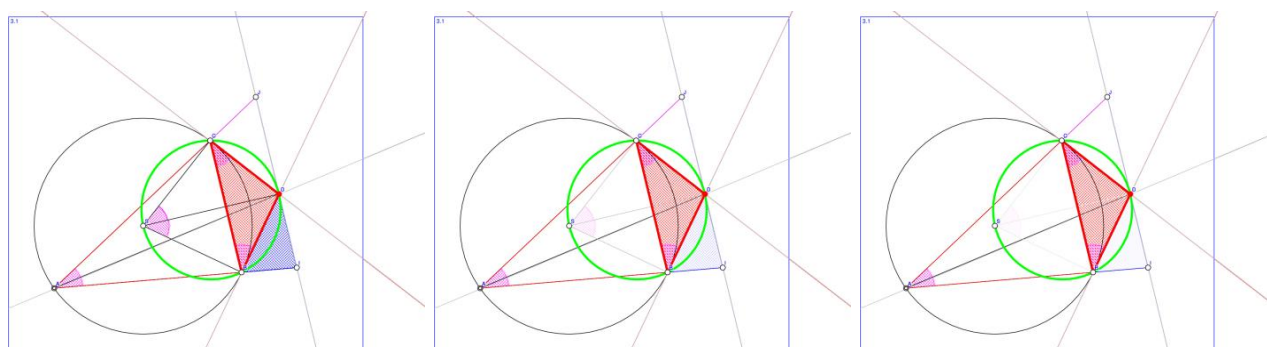


Figure 41

Transparent objects are transparent (invisible) only in icons (they appear yellow in Sketch editor). Object can be transparent in some scenes and visible in other scenes of the same icon. There is an important distinction between hidden and transparent objects: transparent objects are considered in observation while hidden objects are not.

There are two available types of auxiliary objects in OK Geometry. The first type is the **text object** (simple text in the drawing), which can be used to clarify a construction or for other purposes. The second one is the **marker object**, which consists of a symbol (①, *, ?, etc.) and a text that is only visible when the mouse pointer is positioned on the symbol (Figure 42). Markers are used for different types of comments to be selectively visible.

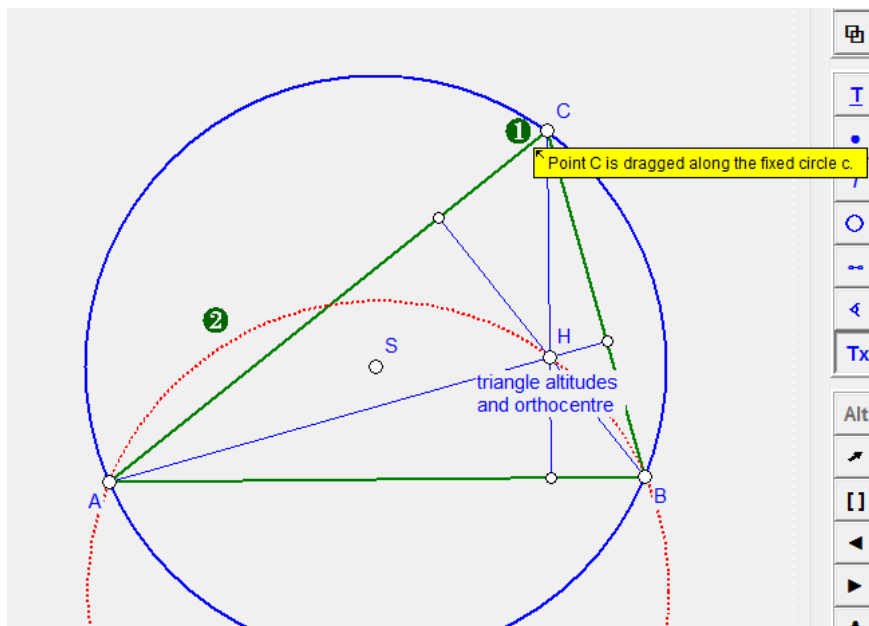


Figure 42

In the Project view the emphasis, highlight, and bleach are controlled with the sliders in the left pane. Elsewhere the buttons in the upper right part of the display can be used (Figure 43).

If button **H** is active, hidden objects are displayed as if they were not hidden.

If button **B** is active, bleached objects are displayed as if they were not bleached.

If button **T** is active, texts in the drawing are visible. However, labels of objects are still visible.

If button **M** is active, markers visible otherwise they are withdrawn.

If button **N** is active, transparent objects are visible otherwise they are not.

if button **X** is active, then auxiliary lines (related to angles) are visible.

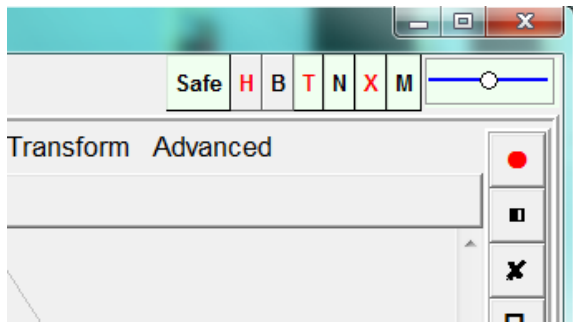




Figure 43

10.3 Presenting the work

The commented relations can be presented one by one in the same order as in the list in the left pane. First set the level of details to be presented (in case the icons are organised in detail-levels). Choose level 1 to display only the basic points of the work, and level 3 to display all details. In order to start the

presentation press the  button below the main menu. A big icon appears (Figure 44). To move between the icons use the < and > buttons on the title of the Task section. Use the >> and << buttons to skip the details. Alternatively, use the **PgUp**, **PgDn**, **Home**, **End** keys on the keyboard. During the presentation the Task and Properties section in the left pane can be shrunk or expanded. To stop the presentation, click again the  button.

If we want to present the various features of the construction one by one, we associate the related icon to **scenes**. Each scene of an icon emphasises a feature on the underlying construction of the icon. For each scene it is possible to specify the objects to be emphasised, bleached and/or make transparent. Scenes can be set in Sketch editor (see the Sketch Editor Reference). In the presentation of an icon you can visualise various scenes of the icon, if they were set. To select a scene just click on one of the scene numbers displayed above the visualisation pane.

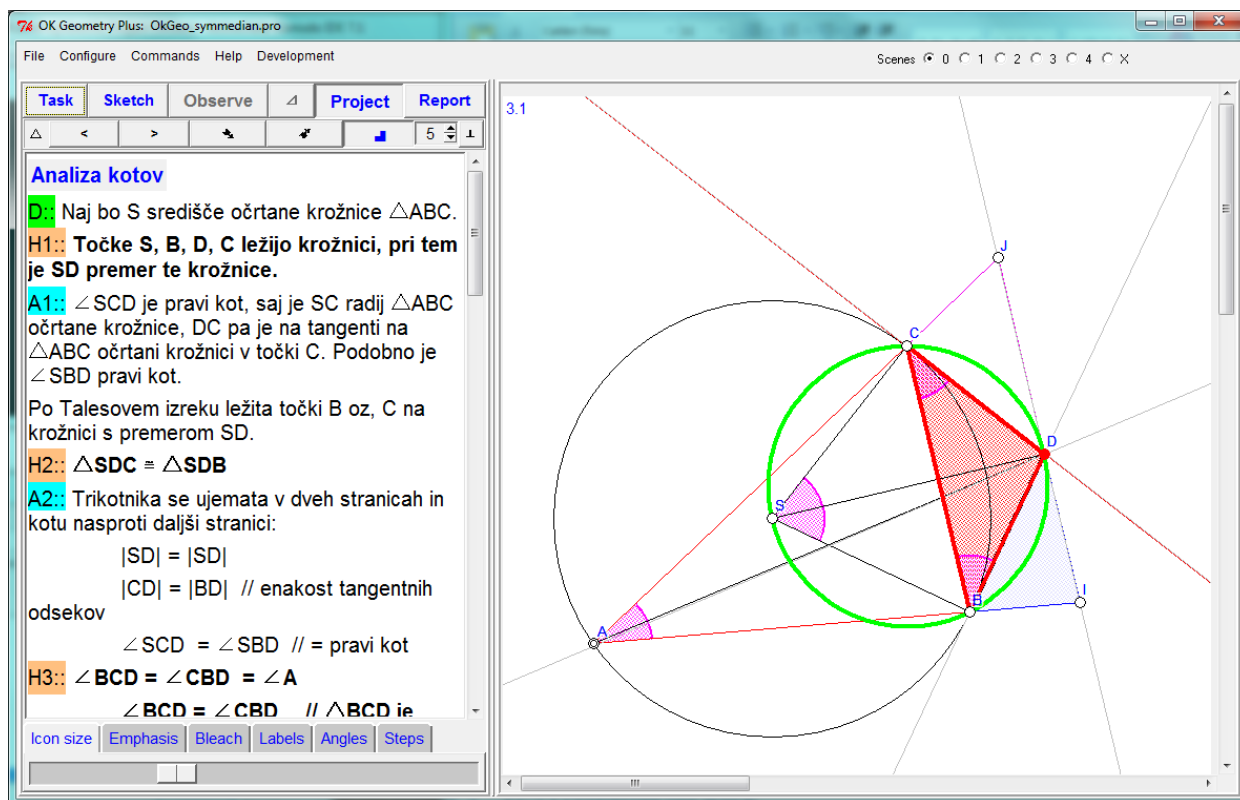


Figure 44

11 Making a report

A report combines some or all icons of a project, together with their comments, into a document, which can be further edited with an appropriate application. Note that if you choose to include in your report figures in PDF, EMF, EPSI format or if the figures contain symbols, the inkscape software should be installed on the computer.²

OK Geometry produces documents in (essentially) three formats:

HTML format (file endings .htm, .html, .mhtml, .mhtml) is best viewed on computer display. Preferred format for figures are PNG and SVG.

DOC format (file endings .doc, .docx) is a standard format for word processors, like Microsoft Word. Preferred format for figures are PNG and EMF.

TEX format (file endings .tex) is a format used in typesetting applications.

HTML and DOC formats allow several report styles. Icons and comments can be arranged in various ways in columns or otherwise (Figure 46).

Here are the steps for generating a report:

1. Activate the Report command (in Project view).

² Inkscape is a free software under GPL license. It is available at <https://inkscape.org/>.

2. Select the name of the file to contain the report. ***The format of the file depends on the file ending of your file.***
3. In the form that appears (select the icons to be included in the report.
4. Click Advanced features and set more the features of the report.
5. Click OK to create the report.
6. Probably, some features In the obtained report turn to be not as desired (Figure 45): point labels are perhaps too small, line are perhaps too thin, etc.
7. Repeat the whole process, but set Advanced options according to resulting features in step 6.
8. Finish the report in an appropriate application.

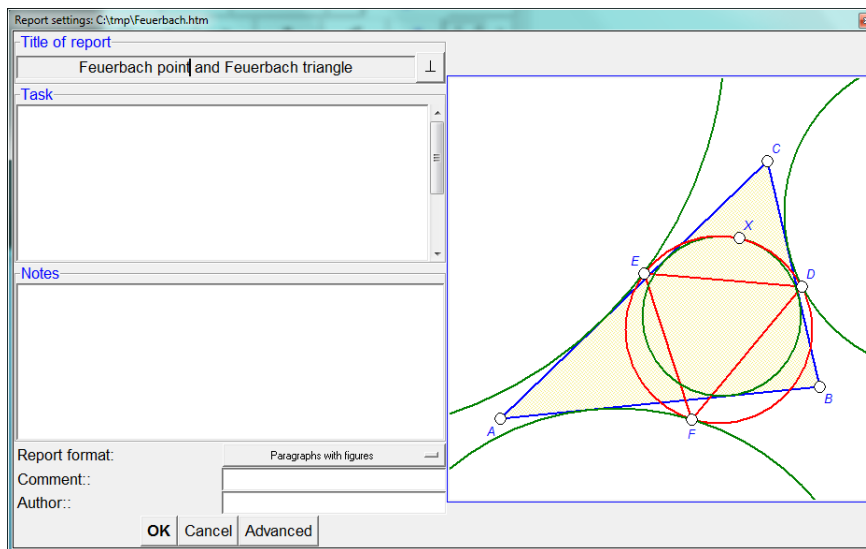
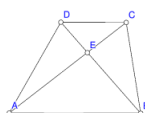
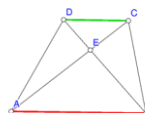
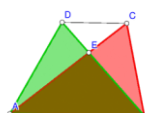
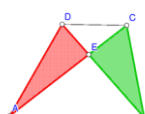
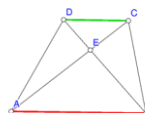
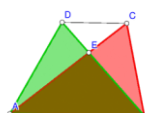
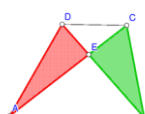
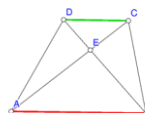
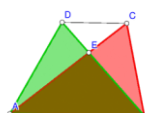
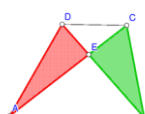


Figure 45

Figure 46 illustrates some available styles of the report.

<div></div> <p>Task</p> <p>1 Task</p> <p>Here is a trapezium ABCD. The point E is the intersection of the diagonals. Prove that the areas of triangles AED and BEC are equal.</p> <p>Comment: Two columns with icons</p> <table><tr><td><div></div></td><td><p>2 parallel lines</p><p>Claim 1 $AB \parallel CD$</p></td><td><p>Argument 1 By definition of trapezium, since ABCD is a trapezium.</p></td></tr><tr><td><div></div></td><td><p>3 same areas</p><p>Claim 1 $\text{area}(\triangle ABC) = \text{area}(\triangle ABD)$</p></td><td><p>Argument 1 Triangles $\triangle ABC$ and $\triangle ABD$ share the same basis AB and have equal heights because of #2.</p></td></tr><tr><td><div></div></td><td><p>4 areas of triangles</p><p>Claim 1 $\text{area}(\triangle ADE) = \text{area}(\triangle BCE)$</p></td><td><p>Argument 1 From the equal areas, see #3, of $\triangle ABC$ and $\triangle ABD$ we subtract the common area of $\triangle ABE$. The remaining parts, $\triangle AED$ and $\triangle BCE$, have the same area.</p></td></tr></table>	<div></div>	<p>2 parallel lines</p> <p>Claim 1 $AB \parallel CD$</p>	<p>Argument 1 By definition of trapezium, since ABCD is a trapezium.</p>	<div></div>	<p>3 same areas</p> <p>Claim 1 $\text{area}(\triangle ABC) = \text{area}(\triangle ABD)$</p>	<p>Argument 1 Triangles $\triangle ABC$ and $\triangle ABD$ share the same basis AB and have equal heights because of #2.</p>	<div></div>	<p>4 areas of triangles</p> <p>Claim 1 $\text{area}(\triangle ADE) = \text{area}(\triangle BCE)$</p>	<p>Argument 1 From the equal areas, see #3, of $\triangle ABC$ and $\triangle ABD$ we subtract the common area of $\triangle ABE$. The remaining parts, $\triangle AED$ and $\triangle BCE$, have the same area.</p>	<p>Task</p> <p>1 Task</p> <p>Here is a trapezium ABCD. The point E is the intersection of the diagonals. Prove that the areas of triangles AED and BEC are equal.</p> <p>Comment: Paragraph with icons</p> <p>2 parallel lines</p> <p>Claim 1 $AB \parallel CD$</p> <p>Argument 1 By definition of trapezium, since ABCD is a trapezium.</p> <p>3 same areas</p> <p>Claim 1 $\text{area}(\triangle ABC) = \text{area}(\triangle ABD)$</p> <p>Argument 1 Triangles $\triangle ABC$ and $\triangle ABD$ share the same basis AB and have equal heights because of #2.</p> <p>4 areas of triangles</p> <p>Claim 1 $\text{area}(\triangle ADE) = \text{area}(\triangle BCE)$</p> <p>Argument 1 From the equal areas, see #3, of $\triangle ABC$ and $\triangle ABD$ we subtract the common area of $\triangle ABE$. The remaining parts, $\triangle AED$ and $\triangle BCE$, have the same area.</p> <p>Notes</p>
<div></div>	<p>2 parallel lines</p> <p>Claim 1 $AB \parallel CD$</p>	<p>Argument 1 By definition of trapezium, since ABCD is a trapezium.</p>								
<div></div>	<p>3 same areas</p> <p>Claim 1 $\text{area}(\triangle ABC) = \text{area}(\triangle ABD)$</p>	<p>Argument 1 Triangles $\triangle ABC$ and $\triangle ABD$ share the same basis AB and have equal heights because of #2.</p>								
<div></div>	<p>4 areas of triangles</p> <p>Claim 1 $\text{area}(\triangle ADE) = \text{area}(\triangle BCE)$</p>	<p>Argument 1 From the equal areas, see #3, of $\triangle ABC$ and $\triangle ABD$ we subtract the common area of $\triangle ABE$. The remaining parts, $\triangle AED$ and $\triangle BCE$, have the same area.</p>								
Two columns with icons	Paragraphs with icons									

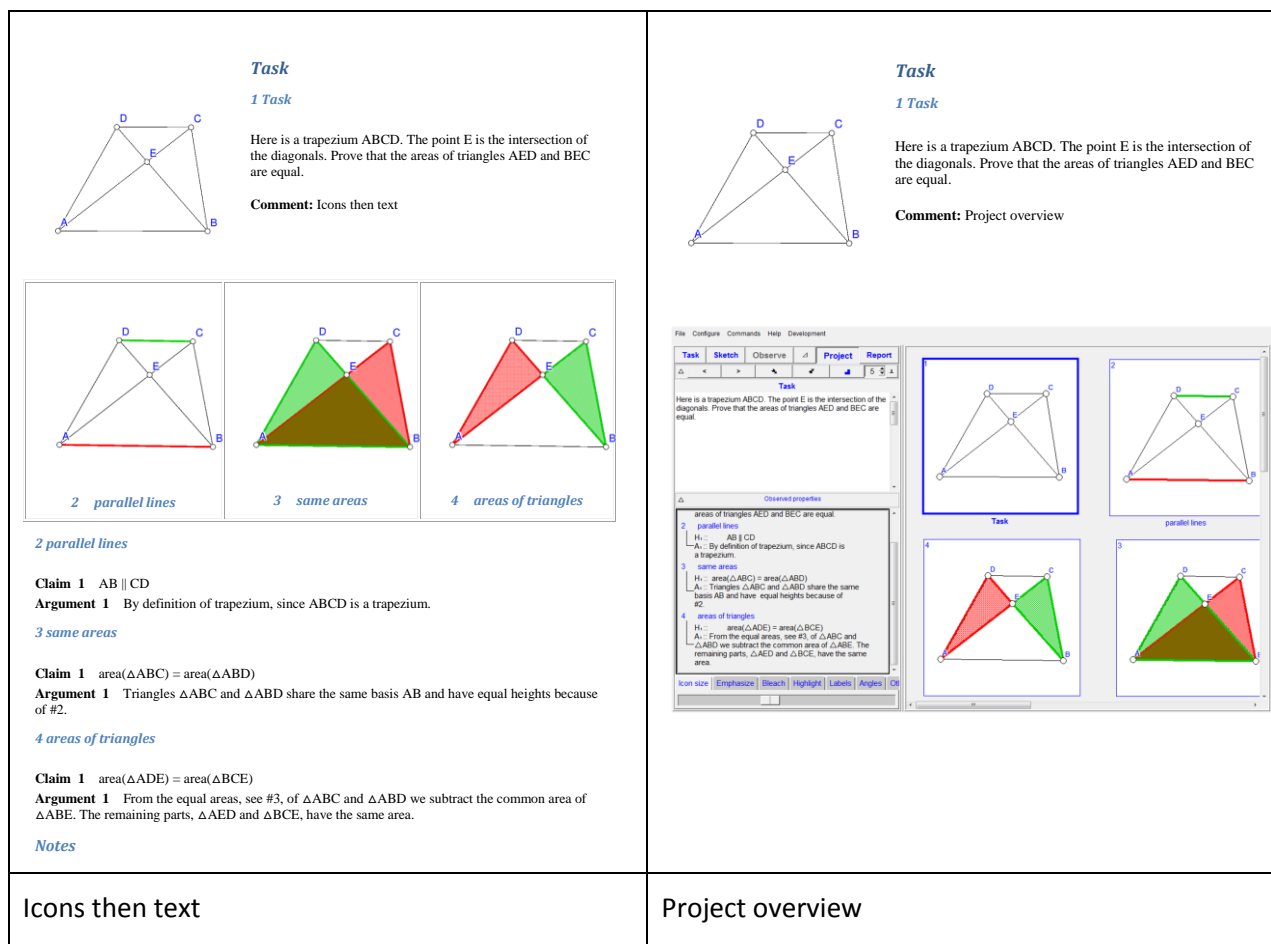


Figure 46

12 Menu commands

Here is a brief overview of the available menu commands:

12.1 File Commands

Command	Meaning	Comments
New project	Begin a new project	Begin a new project. Deletes all icons as well as the current construction and starts the Sketch editor.
Open Open construction	Open (import) an OK Geometry construction file Import a construction from Cabri, Cabri Express, GeoGebra, Cinderella, C.a.R. (Compass and Ruler), Sketchometry, JGEX or OK Geometry file	Read a construction that was saved in OK Geometry or in some other format. The read construction overrides the current construction.
Open Open project	Open a project	Reads a project that was previously saved to a file with .pro extension.
Save Save construction	Save the current construction to a file	The current construction is saved as OK Geometry file with .p extension.
Save Save project	Save the current project	The current project is saved as OK Geometry file with .pro extension.
Save Save	Save the current work to the related .p or .pro file.	
Export Export image	Export image to clipboard, raster file or vectorised file	
Export Export to static GeoGebra construction	Export image to static GeoGebra construction	
Recover last session	Recover last session.	OK Geometry regularly stores the current work. In case of some kind of failure with this command the session can be recovered immediately after the start

		of a new session.
Inspect files	Overview dynamic constructions in various formats	Inspection of files in various dynamic geometry formats with the visualisation of the related constructions.
Exit	Exit from OK Geometry	

12.2 Configuration Commands

Command	Meaning	Comments
Working mode	Choose between the Easy mode, the Basic mode, and the Plus mode of work using OK Geometry	<p>The Easy mode is intended for occasional users. In this mode it is possible only to analyse imported geometric constructions or produce reports (e.g. proofs) from previously schemas. See Help for Simple mode for details.</p> <p>The Basic mode is intended for users who want to have full control over OK Geometry when studying geometric situations. The Basic mode encompasses a powerful geometry editor (see Sketch Editor Reference).</p> <p>The Plus mode is intended for specialist in triangle geometry. In this mode some additional functions are included, and a wider range of properties are studied.</p>
General options	Set the general configuration data	Here it is possible to set several working options (language, size of fonts, etc.).
Considered properties	Information about the properties that are checked for various levels of analysis.	Select the level you are interested in to display the checklist of the properties.
Load configuration	Load a previously saved configuration of OK Geometry	A configuration of OK Geometry can be stored and retrieved.
Save configuration	Save the current configuration of OK Geometry	A configuration of OK Geometry can be stored and retrieved.

12.3 Workflow Commands

This menu contains mainly commands from the main menu bar and some special commands available only in the Plus mode.

Command	Meaning	Comments
Conform icons	<p>Transfer selected properties of an icon to selected icons.</p> <p>This command also adjusts the size of the represented word and/or the represented objects of the selected icons so that they conform with those of a given icon.</p>	<p>This command also adjusts the size of the represented word and/or the represented objects of the selected icons so that they conform with those of a given icon.</p>
Slightly shake the construction	<p>Slight modification of the current construction (F6)</p>	<p>The free points in the current dynamic construction are slightly moved. The movements are minor so that the construction remains essentially the same.</p>
Construction steps	<p>Display construction steps and scenes of the current construction</p>	

12.4 Help Commands

The meaning of menu entries is self-evident.