

OK Geometry Plus

Reference for OK Geometry Plus (v.18.1)

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1. Introduction

This document describes specific features of OK Geometry Plus. OK Geometry Plus is intended for advanced users interested in triangle geometry. Several features that are added to the Basic module of OK Geometry allow constructions based on advanced geometric properties of triangles and triangle related operations. Moreover, these features and operations are used in the (observational) analysis of geometric configurations.

The Plus mode of OK Geometry can be accessed with the command *Configure/Working mode/Plus* in the main menu.

The core of OK Geometry Plus is a large database of

- triangle centres (e.g. symmedian point, currently more than 30 000 of them),
- triangle related triangles (e.g. orthic triangle, currently more than 200 of them),
- triangle related lines (e.g. Euler line, currently 11 lines, more than 2000 including those of related triangles),
- triangle related circles (e.g. 9-point circle, currently 36 circles, more than 5000 including the characteristic circles of related triangles),
- triangle related conics (e.g. Feuerbach hyperbola of a triangle, currently 18 conics),
- triangle operations that associate a point, line, circle, triangle, or conic to a given point (e.g. isogonal conjugation of a point, currently 49 operations).

The database elements can be used directly in constructions as well as in observation of objects. The Plus mode comprises three main procedures:

- Simple observation of triangle objects
- Constructions with various triangle related objects, cyclic constructions
- Advanced observation of triangle objects

In the final chapter we present some examples.

Triangle geometry involves a multitude of concepts (various objects and operations). In order to facilitate the work, a help system was developed. It is explained in the section that follows.

2. The help system

The help system of OK Geometry Plus consists of two tools: *Triangle commands* and (triangle) *Glossary*. Both tools are accessible in all triangle related texts in OK Geometry, including in texts inside the tools themselves.

2.1. The Glossary

The triangle Glossary can be accessed with the command *Help/Glossary* or with the **F1** key. For a given term the glossary provides a lists of concepts related to that term. We explain the use of glossary using an example.

Suppose that we come across the term 'Gergonne point'. We may not know what it is and where to look in OK Geometry for the related commands. We call up the Glossary (by pressing **F1**) and enter the

(approximate) term - for example 'gergone'. In the form a list of about 20 items appears, listed in decreasing relevance:

- first, the articles that contain the searched term (or a similar term) in their title,
- then, the articles (preceded by -) that contain the searched term (or a similar term) in the explanatory text,
- finally, preceded by - - the centres form ETC (Encyclopaedia of Triangle Centres) that contain the term (or a similar term) in the name of the centre.

A click on any article provides the information in that article. In our case we click the first item ('Gergonne point'), and the form is filled, see Figure 1.

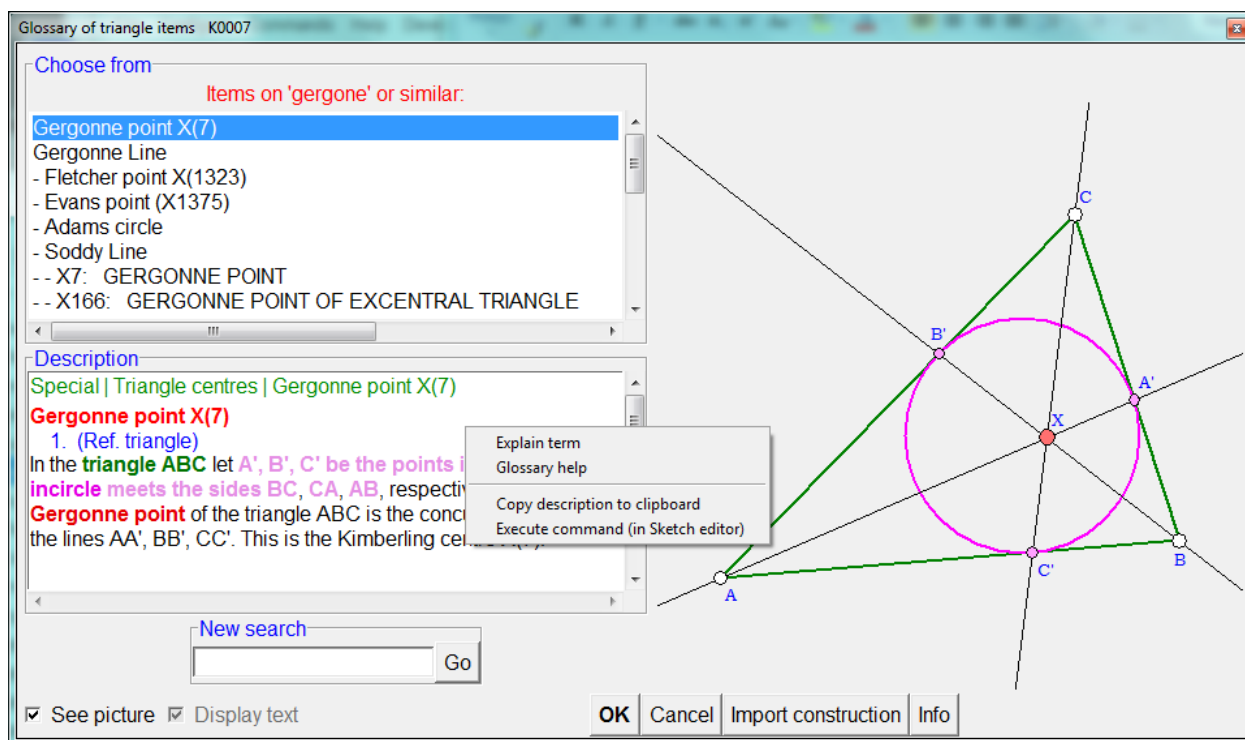


Figure 1

The contains a **description** and an **illustration** of the selected term. Since there is a command related to the Gergonne point, the description also contains the position of the command in the menu (first line) and the arguments of the command.

A right click on the description leads to a menu with further help.

Explain terms turns the cursor into a question mark with an arrow. A click with the arrow on a term in the description (e.g. 'incircle') shows an explanation of the meaning of the term (possibly including the adjacent words).

Glossary help turns the cursor into a spider. A click of the spider on a term in the description activates the glossary for the that term.

Execute command is active only in the Sketch Editor. It activates the form that contains the desired command. In our case to the list of triangle centres where we can select the command *Gergonne point*.

2.2. Triangle commands

The *Help/Triangle commands* displays the list of all commands for the creation of triangle related objects and operations. The list can be restricted to commands that create a specific type of objects. For each command there is a description, which has the same functionality as the description in the glossary. The command can be therefore activated by a right click on the description.

3. Simple observation of triangle objects

Assume a construction has been imported from some dynamic geometry system or constructed in OK Geometry. We describe here how to relate constructed objects to a given triangle – called the **reference triangle**.

We explain this on a simple example (Figure 2). Let A' , B' , C' be the basepoints of altitudes from the vertices A , B , C of triangle $\triangle ABC$. Furthermore, let A'' , B'' , C'' be the midpoints of $B'C'$, $C'A'$, and $A'B'$. By observing the configuration or in otherwise, we find out that the lines AA'' , BB'' , CC'' (apparently) meet in a common point P . We wonder how is this point related to $\triangle ABC$.

If we are not in Plus mode, we need to switch to it – in the menu on top choose the command *Configure/Working mode/Plus*.

Then click on the command  (*Analyse objects in reference triangle*) on the taskbar on right.

If the reference triangle has not yet been defined, first enter the labels of points that determine the reference triangle (Figure 3). The reference triangle is coloured pale yellow. You can declare the reference triangle at any time with the command *Comands/Set reference triangle*.

Then click on the objects about which you want to know something, in our case the point P .

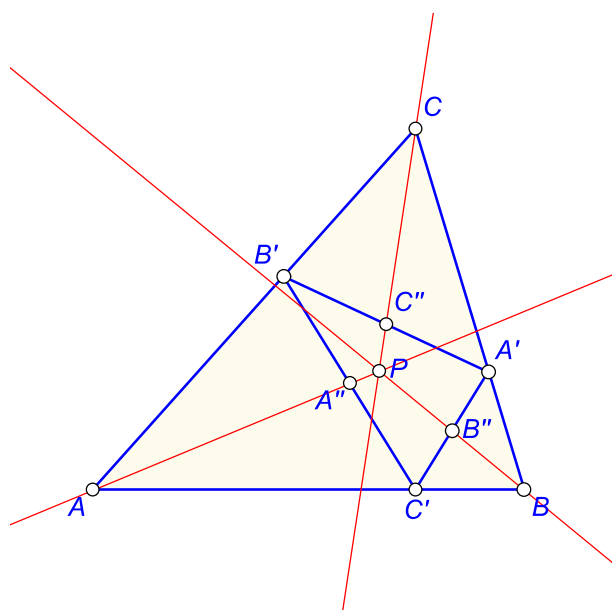


Figure 2

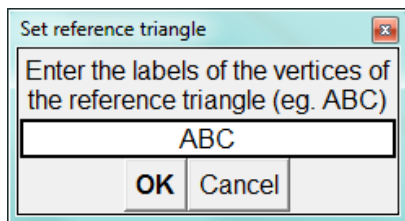


Figure 3

After a short calculation a form appears with the observational results ().

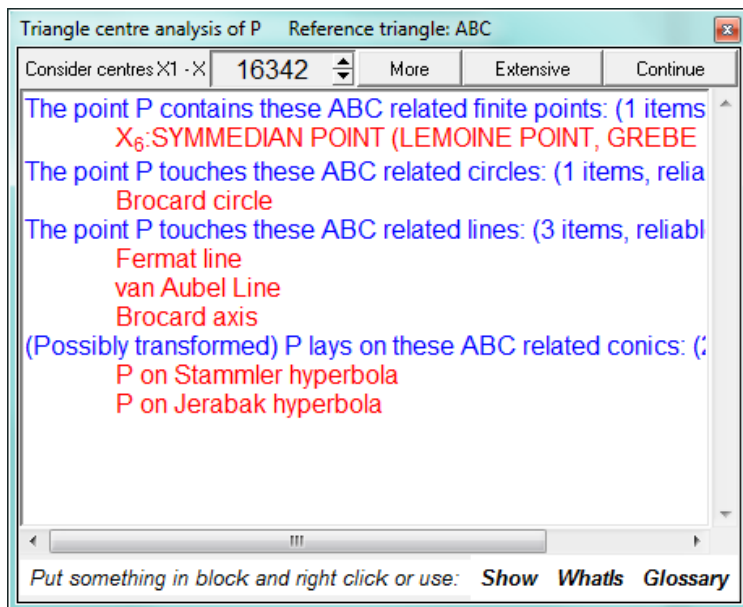


Figure 4

OK Geometry observes that the point P appears to be the symmedian point (commonly denoted as X6) and presents some additional observed facts regarding the point P. To view more information about the same object, click the *More* or *Extensive* button.

The listed objects can be readily displayed. For example, to display the Fermat line, put the text Fermat line in a block (or simply position the cursor on the text) and click the **Show** command – the Fermat line appears on the construction (Figure 5). To add more objects from the list to the construction at the same time, simply put the objects in a block and click the **Show** command.

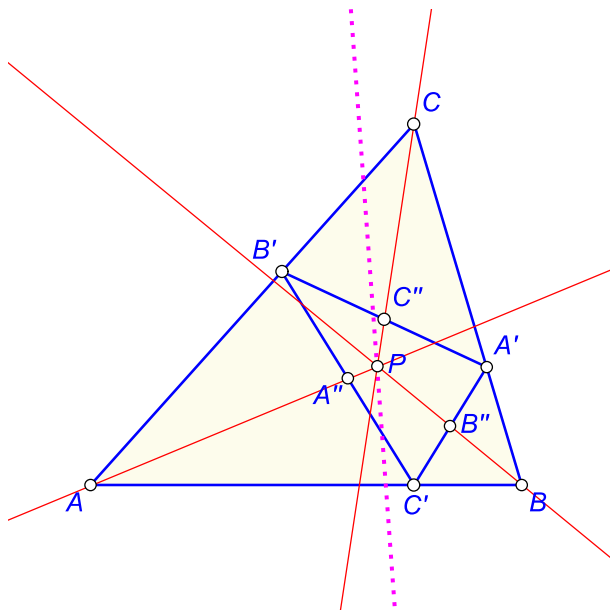


Figure 5

In a similar way an explanation of the listed terms can be obtained. For example, to obtain some basic information of the Fermat line (or some other term), click the **WhatIs** command and click the term with the cursor (question mark with arrow). An explanation of the Fermat line is shown (Figure 6).

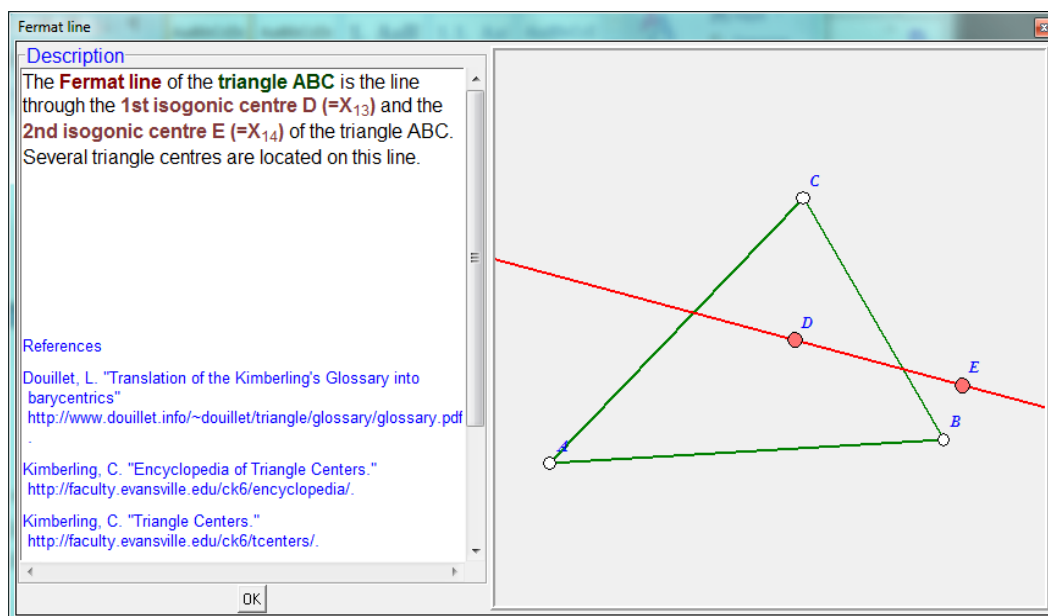


Figure 6

Finally, a glossary that relates a term to other similar term, used in triangle analysis, is provided. For example, to find articles related to symmedian point, click the **Glossary** command and then click the term Symmedian point with the cursor (a sider). The list of articles related to symmedian appears (Figure 7).

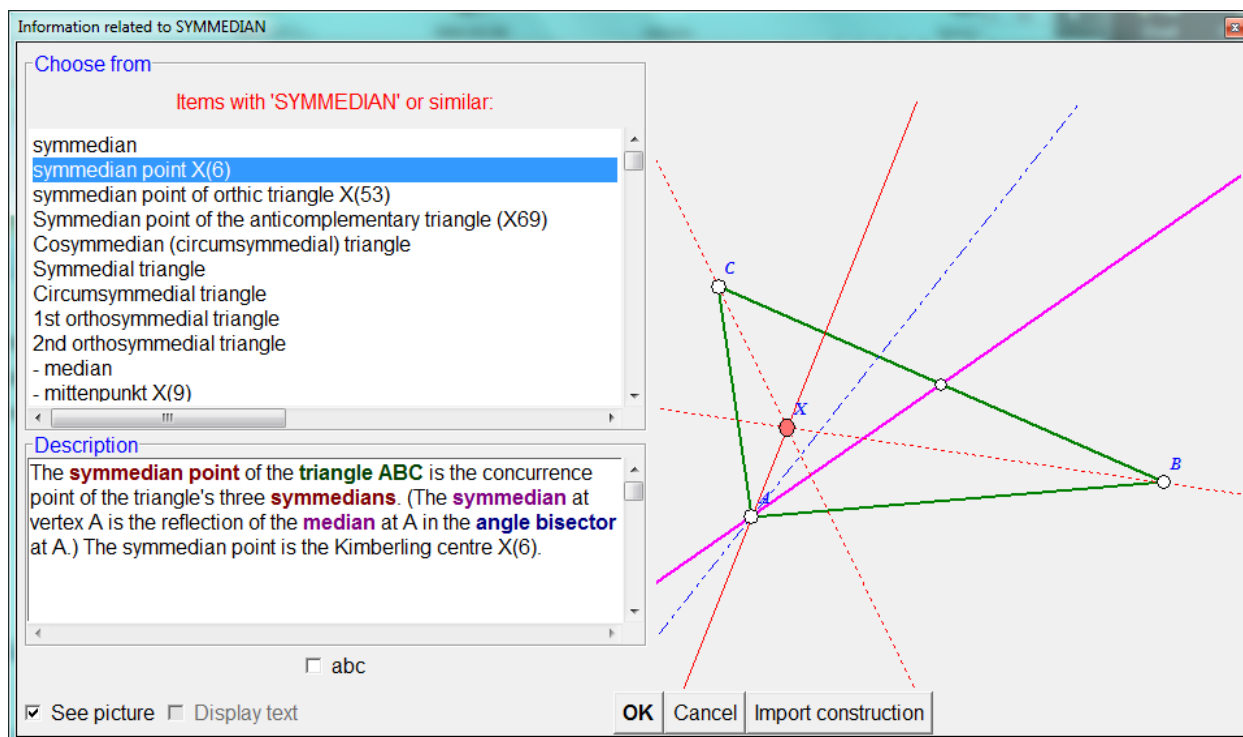


Figure 7

Some additional commands related to the form (Figure 4) are available by a right-click on the form.

In the form (Figure 4) we can also set the range of points (from the ETC list of triangle centres¹) to be used in the analysis. Complete analysis can consider the first 16000 points, a somehow restricted and less reliable analysis can take account of more than 30000 points.

4. Special commands

The Sketch editor In Plus mode has an additional group of commands (**Special commands**). All commands in this group are specific to triangle geometry and most of them have a similar syntax.

The special commands enable the construction of various objects related to a triangle. The triangle to which we refer to is hereby called the **reference triangle**. As the same triangle is often used repeatedly as a reference triangle, it is common practice to declare a default reference triangle for most of the commands in the group. This is done with the command *Commands/Set reference triangle* or *Special/Set reference triangle*. The commands in this group refer to the default reference triangle; however, it is always possible to specify a different reference triangle *as the first argument of the command*. Figure 8 shows a typical group of commands (characteristic objects of a triangle). Note the red line with the checkmark at the bottom, which indicates that the default reference triangle will be used unless the checkmark is removed. Also note also the checkmark *See picture* at bottom. When this checkmark is set, a pictorial representation of the object appears on the right.

¹ ETC is acronym for Encyclopedia of Triangle Centers (<https://faculty.evansville.edu/ck6/encyclopedia/ETC.html>).

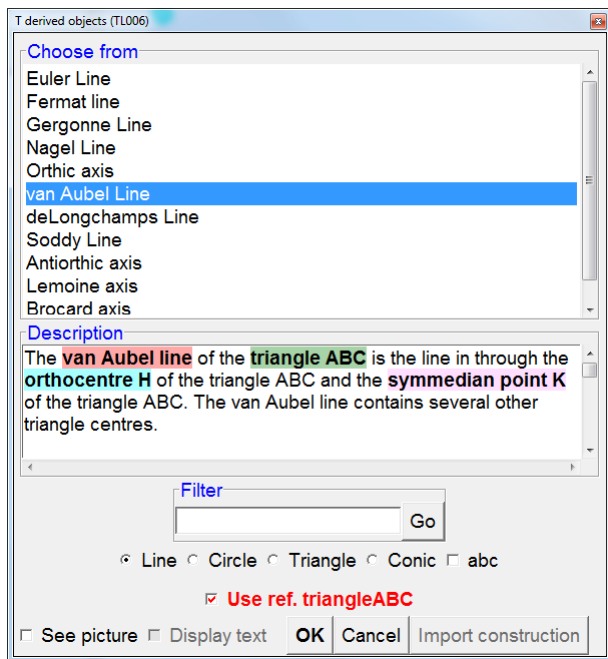
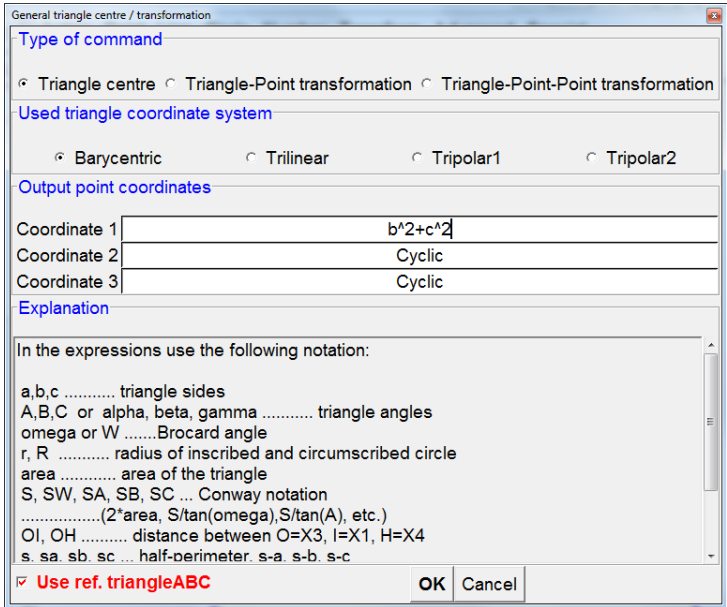


Figure 8

The Special commands menu the commands are grouped according to the input objects:

<p><i>Triangle centres</i></p>	<p>Triangle centres are organised in five lists:</p> <ul style="list-style-type: none"> - simple centres - the four classic triangle centres; - advanced centres - the most known centres, e.g. isodynamic point; - ETC centres - the first 16341 centres in the ETC list of triangle centres; - ETC+ centres - the centres from 16342 on in the ETC list of triangle centres; - bicentric - some well known triangle bicentres, e.g. Brocard points. <p>A standard notation is used for the centres from the ETC list: X_n, X_n or $X(n)$ for the n-th centre. The centre X_0 has a special meaning – it is the variable centre, which can take place of any centre.</p>
<p><i>Triangle derived objects</i></p>	<p>The classic triangle objects, other than points, are organised in several lists:</p> <ul style="list-style-type: none"> - triangle lines, e.g. the Euler line; - triangle circles, e.g. the Brocard circle; - triangle triangles, e.g. the orthic triangle; - triangle conics, e.g. the Kiepert hyperbola.
<p><i>Triangle/Point derived objects</i></p>	<p>A point is associated to various objects in reference triangle. Such objects are organised in several lists, depending on the type of the resulting objects. The considered type of resulting objects are:</p> <ul style="list-style-type: none"> - points, e.g. the isogonic conjugation of a point in a reference triangle;

	<ul style="list-style-type: none"> - lines, e.g. the trilinear polar of a point in a reference triangle; - circles, e.g. the Cevian circle of a point; - triangles, e.g. the pedal triangle of a point in a reference triangle; - conics, e.g. the inconic of a point.
<i>Triangle/Point/Point derived objects</i>	Here is a list of binary point operations that are defined in a reference triangle, e.g. the cross-conjugation of two points.
<i>Objects by triangle centres</i>	<p>This command allows an effective way to define and visualise triangle related objects passing through specified triangle centres. Simply enter the centres as indicated by the following examples:</p> <p>1-100 The first 100 centres</p> <p>(2,14) The line through X(2) and X(14)</p> <p>(2,14,15) The circle through X(2),X(14),X(15)</p> <p>(5,6,7,8,9) The conic through X(5),X(6),X(7),X(8),X(9)</p> <p>2,4,(3,6) The centres X(2) and X(4) and the line through X(3) and X(6)</p>
<i>General triangle derived point</i>	<p>Here it is possible to analytically define centres, point transformations or point operations in a reference triangle. In the definition barycentric, trilinear or tripolar coordinates are allowed, as well as various triangle parameters.</p> <p>In the example below (Figure 9) in the reference triangle with sides a, b, c we define the point with barycentric coordinates $(b^2+c^2, c^2+a^2, a^2+b^2)$.</p>  <p>Figure 9</p>
<i>Various constructions</i>	Here are collected various constructions of points, lines, circles, and conics with various input objects. We list only some remarkable examples:

	<ul style="list-style-type: none"> - perspectivity centre of two triangles (if it exists); - orthology centre of two triangles (if it exists); - parallelogy centre of two triangles (if it exists); - cyclology centre of two triangles (if it exists); - trilinear pole of a line wrt. A triangle; - perspective axis of two triangles (if it exists); - the perspeconic conic of two perspective triangle. <p>The commands in this group do not make use of the default reference triangle.</p>
<i>Set reference triangle</i>	<p>This command defines the reference triangle, which is (optionally) used by default in other special commands. The vertices of the reference triangle must be labelled.</p> <p>To clear the reference triangle enter in the form an empty line.</p>
<i>A triangle</i>	<p>This command is active only at the very beginning of the construction. It generates a triangle with labelled vertices and cyclic structure.</p>
<i>Declare cyclic objects</i>	<p>Declare three objects (of the same type) as cyclic in accordance to the current cyclic construction. When the Sketch editor is creating a 'cyclic construction' (see the command below), any command that involves one or more cyclic objects is repeated cyclically on the related triads of objects.</p>
<i>Cyclic construction</i>	<p>Turns the Cyclic construction ON and OFF. When ON, the display pointer assumes the form similar to circle and the commands are executed cyclically relative to the reference triangle.</p>
<i>Analyse object in reference triangle</i>	<p>The command provides information about the triangle centres that lay on a given object. Simply pick a point, a line, a circle or a conic. If a reference triangle is not yet defined, you first specify the reference triangle. Then click an objects, after a while a list of observed properties of the object relative to the reference triangle appears. Click the More or Extensive button for a longer or an extensive list of observations. To inspect other objects, click Continue and pick the next object to be analysed. Here are some illustrative observations:</p> <ul style="list-style-type: none"> - The picked point is the centre X(42) of the reference triangle. - The picked line is tangent to the nine-point circle of the reference triangle. - The picked point is the intersection of lines X(6)X(9) and X(5)X(7). - The picked line passes through X(21) and the isogonal conjugate of X(35).

	<p>Note. This command is intended for a quick analysis. For a more elaborated analysis, see Section 5.</p> <p>Note. Right click on the displayed results: there are several available commands for visualising the listed objects and getting information about them (place the cursor set a block in the text appropriately).</p>
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All commands for triangle centres and other objects, that refer to a triangle, are used in a similar way. Suppose we are looking for the circumcevian triangle of a given point with respect to the reference triangle. Since our object is defined by the reference triangle and a point, choose the **Triangle/Point derived objects** command. In the form that appears (Figure 10), first select the 'Triangle' option (since the resulting object is a triangle). From the operations listed, choose Circumcevian. **Notice the option in the lower left corner: If a reference triangle was previously defined, specify whether you want to use the already defined reference triangle or another triangle.** As you can see, the form contains a short description of the operation with optional illustration (option 'See').

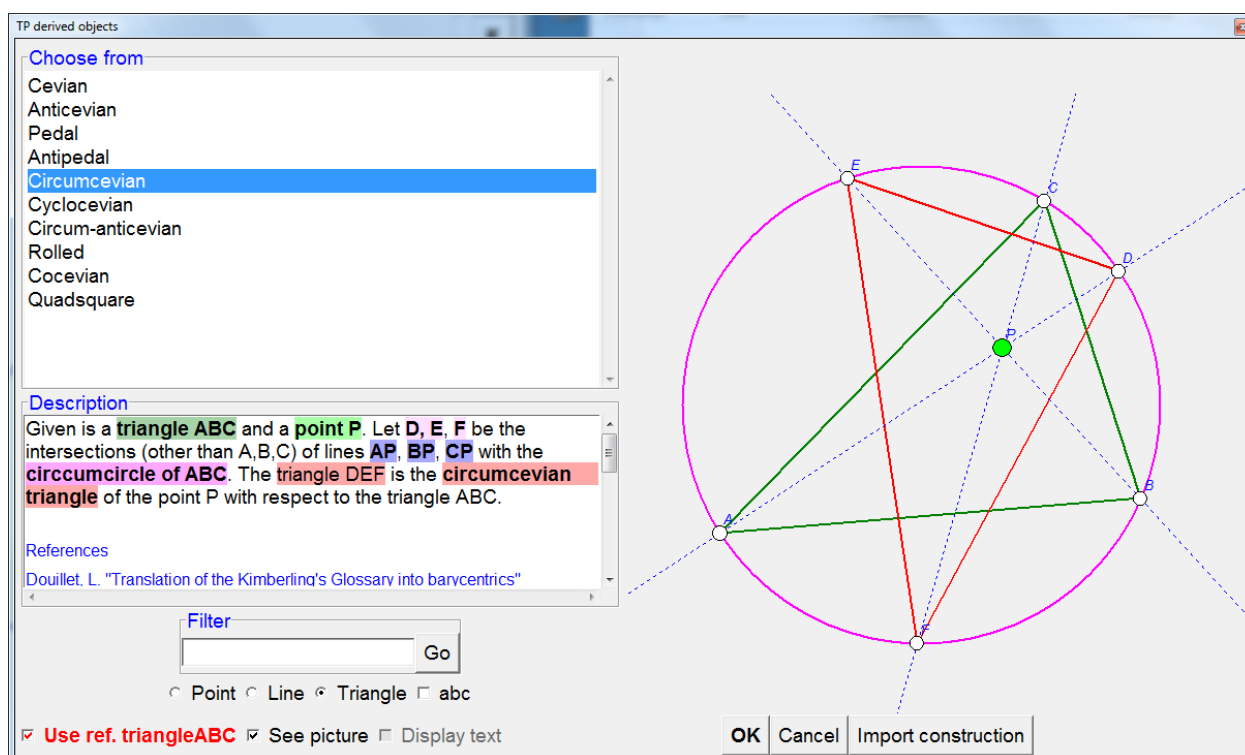


Figure 10

Here is a detailed description of the steps to follow:

1. Select the type of the resulting object (in the above case choose between Point, Line, and Triangle). A list of available objects/operations appears. Select '**abc**' option if you want the list to be ordered alphabetically.
2. Select the object or operation from the displayed list. A short description of the object/operation appears. If you want an illustration of the description, select the '**See**' option.
3. If a reference triangle has previously been defined, you can use it as a reference triangle for the operation.

4. Click OK and select the involved objects:	
<i>If 'Use ref. triangle' is ON</i>	<i>If 'Use ref. triangle' is OFF or is not present</i>
<ol style="list-style-type: none"> 1. Since the reference triangle already defined, you only need to select the additional arguments (depending on the command, there can be none, one point or two points). 	<ol style="list-style-type: none"> 1. ALWAYS specify the reference triangle first. Either pick its three vertices OR pick the polyline consisting of the of the triangle. 2. Select the additional arguments (depending on the command, it can be none, one point or two points).

5. Cyclic constructions

Cyclic construction is an effective way of executing constructions that are cyclic relative to the vertices and sides of a triangle. We illustrate the way of carrying out a cyclic construction using a simple example, the Napoleon point of a triangle.

The (outer) Napoleon point of the triangle ABC is constructed as follows: Place an equilateral triangle outward on each side of the triangle. The line segments connecting each vertex with the corresponding centre of the equilateral triangles on the opposite sides meet at a common point, the (outer) Napoleon point of ABC.

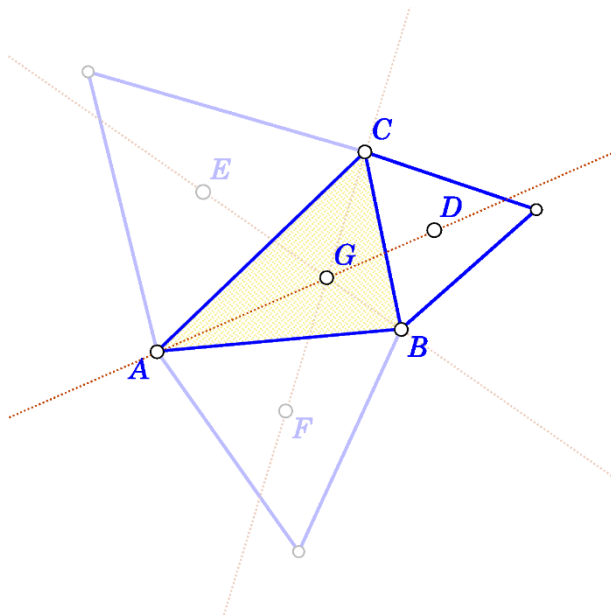
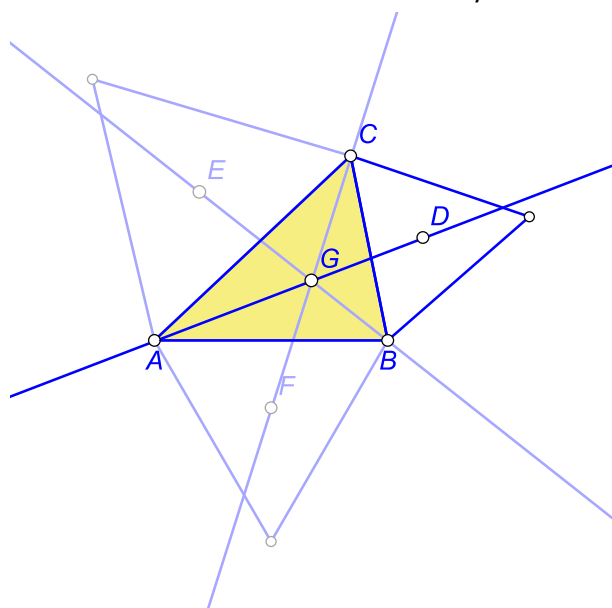


Figure 11

Here is how to do the construction as cyclic construction.



Step/Command	Comment	
Advanced/Shapes/Triangle	Construct a triangle.	All this can be done with single command Special/A triangle.
Actions/Label/Auto label	Label the triangle’s vertices as A, B, C.	
Special/Reference triangle	Declare the triangle ABC as reference triangle (which becomes pale yellow).	
Special/Cyclic objects	Declare the objects (points) A, B, C as cyclic (wrp. to the reference triangle).	
Special/Cyclic	Cyclic construction is turned ON. (Note that the pointer assumes a circular form.)	
Advanced/Shape/Equilateral triangle	Construct the equilateral triangle on the side CB (click vertex C, then B). Simultaneously, bleached equilateral triangles are automatically constructed on the other two sides of the triangle ABC.	
Special/Triangle centres/incentre	Construct the incentre D of the equilateral triangle with CB as side. Be careful not to check ON the ‘Use ref. triangle’ checkmark, since we are not constructing the incentre of ABC. Incentres of the other two equilateral triangles are automatically constructed.	
Actions/Label/Auto label	Label the remaining centres of equilateral triangle as E,F. Note. Labelling vertices is never considered a cyclic operation.	

Line/Line segment	Construct the segment AG. Simultaneously two other cyclically defined segments (BE and CF) are constructed.
Special/Cyclic	Cyclic construction is turned OFF. This is necessary since command that follows must not be applied cyclically..
Point/Intersection	Construct the intersection point G of segments AG and BH. It was necessary to rotate the Cyclic construction OFF as otherwise three intersection points (at the same position) would be constructed.
(Action/Emphasise/Unmark all bleach)	Remove the bleach attribute from all objects, if you wish.

6. Triangle analysis module

The Triangle analysis module is an advanced observation tool for objects in relation to a triangle. We often wonder how a given point, line, circle or conic relates to a given triangle. The Triangle analysis module provides reliable hypotheses about which ETC centres or certain transformation of ETC centres lay on a given line or conic and how given points, lines, circles are related to many of the common objects of the reference triangle.

Let us assume that we come across points P, Q, R when examining a reference triangle ABC. For some reason, we would like to know a little more about these points and about the line PQ. **Triangle analysis** reliably observes properties such as:

- P is the centre X_{541} of ABC, or that P is the cyclocevian conjugation of X_{541} in ABC, or that the complement of P in ABC is X_{541} , or that the Ceva Q conjugate of P in ABC is X_{541} ,
- P is the intersection of lines $X_{17}X_{541}$ and $X_{16}X_{125}$,
- P lays on the Euler line, or that the isogonal conjugation of P lays on the Kiepert hyperbola,
- the line PQ is tangent to the Spieker circle of ABC, or that P is the inverse of X_{17} in the nine-point circle of ABC,
- the circle through P,Q,R contains X_{541} and the isogonal conjugation of X_{130} in ABC.

Triangle analysis can also compare triangle centres of two triangles (e.g. observes that centre X_{541} of ABC is the same point as X_{54} of PQR).

The use of triangle analysis module is rather straightforward: Given a construction, simply specify which objects to analyse with respect to which reference triangle.

6.1. Accessing the Triangle analysis module

To access the Triangle analysis module, click on the triangle (Δ) **symbol in the main menu bar**. Note that for a given construction the analysis can be done only on objects that can be identified by labelled

points on them. In particular, the vertices of the reference triangle should be labelled, and the points, lines, circles, conics to be analysed should respectively contain 1,2,3,5 labelled points.

6.2. Triangle analysis scheme

The best way to learn how to use triangle analysis is by using it. For this purpose some simple and advanced examples are provided (see Section 7). You can learn how to use the triangle analysis module by working out these examples step by step. However, for the sake of completeness we provide here a description of the form that is used in the module.

Task	Sketch	Observe	Project	Report
Type of analysis <input checked="" type="radio"/> Object analysis <input type="radio"/> Variable centre analysis <input type="radio"/> Two triangle analysis		Options Extended treatment <input type="checkbox"/> Perspectivity centres <input type="checkbox"/> Short centre names <input type="checkbox"/> Report size <input type="button" value="Short"/>		
Data for analysis Considered object(s) <input type="text"/> Reference triangle <input type="text"/> Additional ref. points <input type="text"/> Consider centres X1 - X <input type="text" value="3500"/> Variable centre or parameter range <input type="text"/>		Observed results <div style="border: 1px solid black; height: 150px;"></div>		
<input type="button" value="Exit"/> <input type="button" value="Study"/> <input type="button" value="Clear"/> <input type="button" value="AddToNotes"/> <input type="button" value="Show"/> <input type="button" value="WhatIs"/> <input type="button" value="AddToTask"/>				

Figure 12

The following description refers to Figure 12, which shows the form that appears on the left-side pane during the Triangle analysis. Note that, depending on the construction, some of the fields may be missing or may be disabled. Here is a brief description of the meaning of the entries:

Entry	Description
<i>Considered objects</i>	<p>In the entry, write objects that you want to consider in the analysis. Separate the objects with a space or comma.</p> <p>Refer to points as they are labelled, e.g. A' or B or C3, refer to lines by specifying two points on them, e.g. A'B, and refer to circles by specifying three points on their circumference, e.g. A'BC3. Refer to conics by specifying 5 points on them, e.g. A'BC3DE. Labels of vertices with several letters should be put in brackets, e.g. A(PQ3)B.</p>
<i>Reference triangle</i>	<p>Specify the reference triangle by naming its vertices, e.g. ABC. All centres and triangle object in the analysis will refer to this reference triangle.</p> <p>Note. Avoid reference triangles that are isosceles or almost isosceles. In such triangles, many centres are unacceptably close together, so that OK Geometry refuses to perform the analysis.</p> <p>Note. It is possible to specify two or more reference triangles (separated by spaces or commas, e.g. ABC D1E'F). In this case , a sequence of analyses is performed, one for each specified reference triangle.</p>
<i>Additional reference points</i>	<p>The analysis compares the position of a considered object with the position of the reference points and with the characteristic objects of the reference triangle. By default, the reference points are the triangle centres of the reference triangle and, optionally, various transformations of the triangle centres. Here we can specify the additional points to be added to the reference points (i.e. the centres of the reference triangle) .</p>
<i>Condition parameter</i>	<p>This entry appears only if the construction contains a check of a geometric condition (e.g. that three points are collinear). Such situations occur, for example, when the studied construction is based</p> <ul style="list-style-type: none"> - on an implicit construction, or - on optimisation of a parameter, or - on the variable triangle centre X0. <p>If we specify here the condition, the analysis will be done only if the condition is true.</p>
<i>Considered centres</i>	<p>Here we specify the highest index of ETC triangle centres to be considered in the analysis. Note that the when the maximal index is chosen the analysis is less reliable and not all tests are performed.</p> <p>By reducing the number of considered centres, we avoid extensive results list with less known centres. By reducing the number of centres the analysis will be faster.</p>

<i>Variable centre range X</i>	This entry appears only if the construction contains the variable centre X0, which can take the place of any ETC centre. Here we specify, which centres X0 should represent in the analysis. For example, the entry 1-10, 16,17 leads to the analysis of 12 cases where X0 takes the place of X1, X2,..., X10, X16, X17.
<i>Extended tratment</i>	<p>Select this option if you are interested not only in properties of the considered points in relation to the reference points (i.e. triangle centres) but also in properties of transformed considered points in relation to various transformations of the reference points.</p> <p>When this option is selected, you can detect, for example, that a given point is the cyclocevian of X68 or that the complement of your point is the isotomic transformation of X25.</p>
<i>Perspectivity centres</i>	<p>This option is valid only when a triangle is analysed in relation to the reference triangle. The analysis of a triangle looks for various centres of perspectivity (perspective, orthologic, parallelogic, cyclogic) of the considered triangle and various triangles related to the reference triangle. When this option is ON, all the perspectivity centres are calculated and stored, so that they can be accessed and studied later.</p> <p>For example, let $A'B'C'$ be the extouch triangle of the reference triangle ABC. Let us perform the analysis with the option Perspectivity centres ON. after the Study command is pressed, OK geometry reports the observation of 39+92 cases of perspectivity of triangle $A'B'C'$ and various triangles related to the reference triangle ABC. Some of these cases can be essentially the same. These found perspectivities give rise to 198 centres (not all different among them) that are denoted by %1 to %198. The report says, for example, that the triangle $A'B'C'$ is related to the Garcia inner triangle in the following way:</p> <p style="text-align: center;">perspective (%59), orthologic (%60, %61).</p> <p>Thus %59 is the perspectivity centre of $A'B'C'$ and the Garcia inner triangle of ABC, while the point %60 is the orthological centre of $A'B'C'$ and the Garcia inner triangle of ABC, and the point %61 is the orthological centre of the Garcia inner triangle of ABC and $A'B'C'$.</p> <p>To find more about the point %60, simply write %60 as the considered object and study it. The notation %% means all points %1-198 without repetition.</p>
<i>Short centre names</i>	When this option is selected, all the descriptive names of the triangle centres are omitted in the report.
<i>Report size</i>	Here we select the report size which can be Minimal (only the very basic fact are listed), Short, Extended, or All. Note that the option All can give rise to very long lists. It is reasonable to start the analysis with the Short option.

6.3. Observed results

Triangle analysis module runs several types of analysis. The results are always written in the Observed results section in a similar way for all types of analyses. We present here what is possible to do with the results in each type of analysis. In the sections that follow we present illustrative cases for each type of analysis.

1. To run the analysis, click the **Study button**. A report appears in the Observed results section. Figure 13 shows an illustrative example of the beginning part of a report of (the analysed object is a point D).

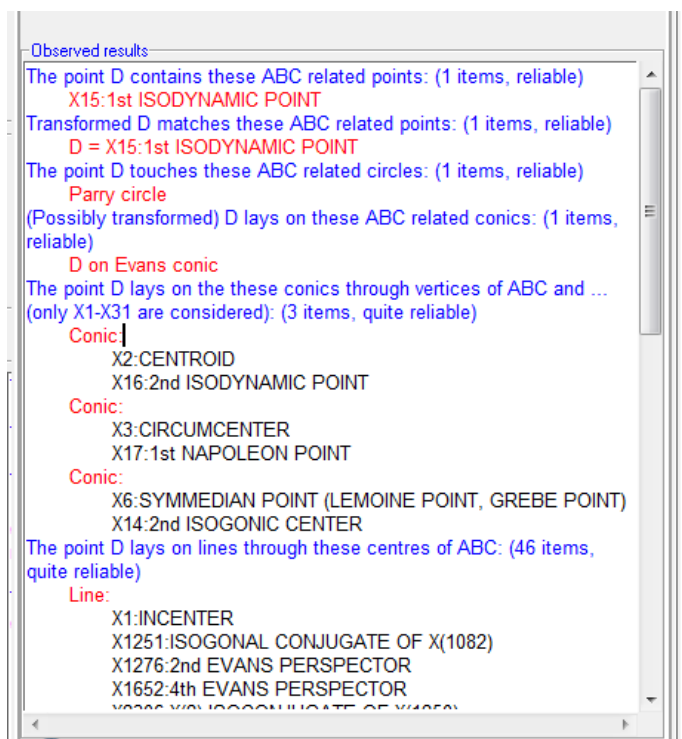


Figure 13

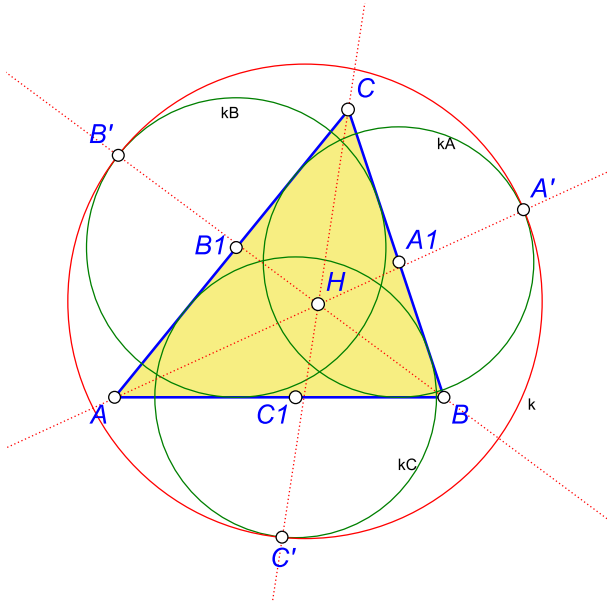
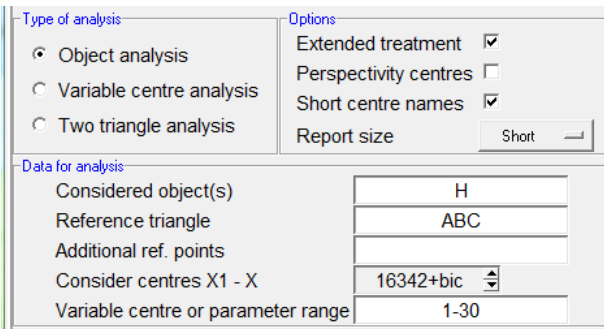
2. The analysis can be run several times in sequence. Simply fill the data, set the options, and press the **Study button**.
3. Reports are always appended to the previous content of the section Observed results. At any time, e.g. before a new analysis is run, you can clear the content of the section Observed results with the **Clear button**.
4. Reports can be freely edited.
5. If you want to include a piece of the Observed results in the OK Geometry report (i.e. the printable report) for the studied construction, put that part in a block and press the **AddToReport button**.
6. To get an explanation about objects that are mentioned in the Observed results section, click the **WhatIs button**. The cursor assumes the form of a question mark with an arrow. Click with the arrow a term in the Observed result section provides and explanation of that term (sometimes the nearby words are also considered). For example, to find out the definition of the Parry circle or of X_{17} , press the **WhatIs button** and put the cursor on 'Parry' or on ' X_{17} '.
7. To visualise the objects that are mentioned in the Observed results section, position the cursor on the name of the object to be shown or put in a block the objects to be shown. Then press the **Show button**. For example, to visualise X_{16} , position the cursor on X_{16} and press the **Show button**. To

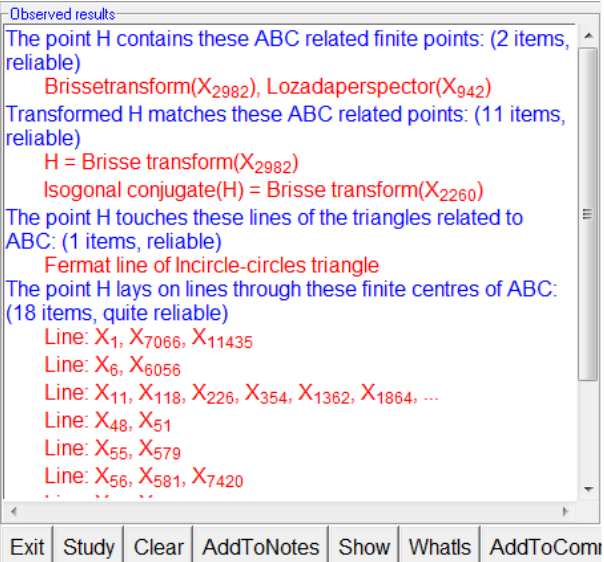
visualise the conic through X_2 , X_{16} , and the vertices of the reference triangle, put in block the three lines from Conic to X_{16} , and press the **Show button**.

8. The **Glossary help** is accessible from the context menu. It turns the cursor into a spider. A click of the spider on a term in the Observed results section activates the glossary for the that term.
9. Finally, to exit from the Analysis module, press the **Exit button** or press again the (Δ) button in the main menu bar.

6.4. Object analysis

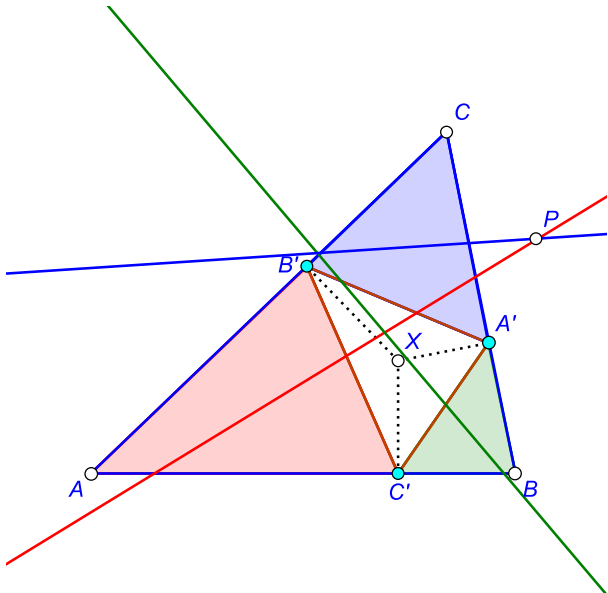
The Object analysis of the Triangle analysis module observes the relationship between constructed objects and the centres and various characteristic objects of a given reference triangle. The essential data for the Object analysis are the reference triangle, which is named after its vertices, e.g. ABC, and one or more objects to be examined. The objects under consideration are identified with vertices on them: one vertex labels refers to points (e.g. P), two vertex labels to lines (e.g. PQ), three vertex labels to circles and triangles (e.g. PQR), five vertex labels to conics (e.g. PQRST). By filling the **Considered objects** entry with 'P Q PQR' we run the analysis for P and Q, the circle through P,Q,R and the triangle PQR.

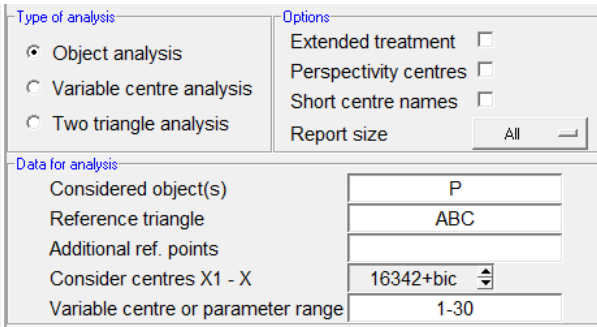
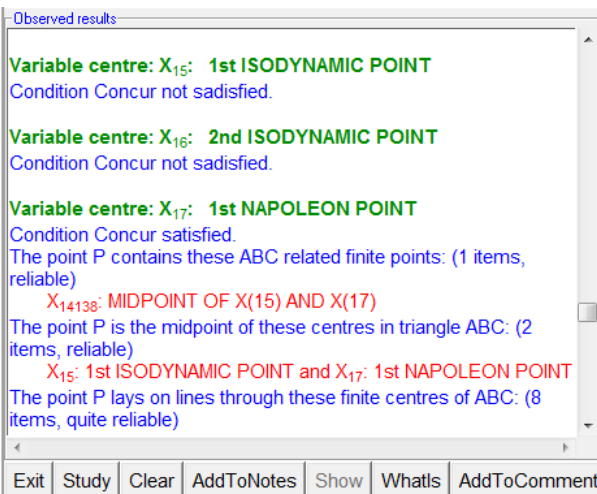
	<p>Given is a triangle ABC. Let k_A be the circle that touches the baselines AB and AC and has the centre A_1 on the sideline BC. Define B_1, C_1 and k_B, k_C cyclically.</p> <p>Let the circle k touch internally the circles k_A, k_B, k_C in points A', B', C'. OK Geometry observes that the lines AA', BB', CC' meet in a common point H.</p> <p>We analyse the point H in relation to the reference triangle ABC.</p>
	<p>Make sure that the Type of analysis is set to Object analysis.</p> <p>Select Short centre names if you prefer a compact report without explanatory names of triangle centres.</p> <p>Select Extended treatment if you want the transformations of H to be included in the analysis.</p> <p>It is advisable to first select a Short Report size</p>

	<p>with Extended treatment OFF. Then, if needed, increase Report size (more instances are displayed) and/or select Extended treatment (additional properties are considered).</p> <p>Press Study to run the analysis.</p>
	<p>The observation finds that H is, for example, the Lozada perspector of X(942), that the isogonal conjugate of H seems to be X(2260), that H is the intersection of lines X(1)X(7066) and X(6)X(6056), etc.</p> <p>To get an explanation for the Lozada perspector, click WhatIs first and then the term Lozadaperspector.</p> <p>To visualise object from the report, put them in block and click the Show command.</p> <p>More available functions are described in section 6.3.</p>

6.5. Object analysis for variable triangle centres

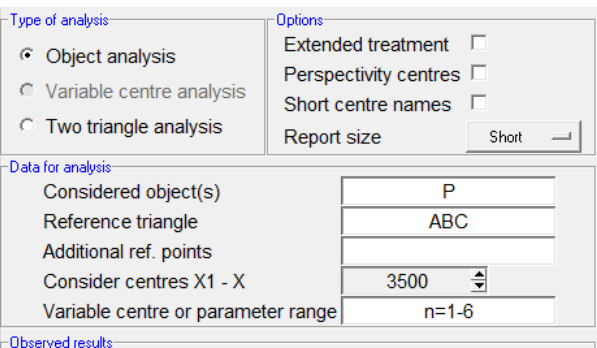
The following example illustrates how to run an observation for a series of constructions in which a point assumes the position of different triangle centres.

	<p>Let $A'B'C'$ be the pedal triangle of a triangle centre X in the reference triangle ABC. Consider the three (shaded) triangles $AC'B'$, $BA'C'$, $CB'A'$ and their corresponding Euler lines. Let us explore, for which centres X ($X=X1, \dots, X30$) the three Euler lines concur in a point P; in case they concur, how is P related to the reference triangle ABC?</p> <p>In the construction we set X as the variable centre $X0$ of ABC (available with the command <i>Special Triangle centre ETC X0</i>). We use the command <i>Special Triangl/Point derived objects Triangle Pedal triangle</i> for the pedal triangle and the command <i>Special Triangle derived objects Line Euler line for the Euler line</i>. Be careful to set the correct reference triangle in the commands.</p> <p>We define the condition named Concur for the</p>
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	<p>concurrency of the three Euler lines and we denote by P the intersection of two of the three Euler lines. The point P is the intersection of the three Euler lines only if the condition Concur is satisfied.</p> <p>Then we switch to the Triangle analysis module.</p>
	<p>Here how we fill in the data. We select the Object Analysis. For the first run it is advisable to choose the Short or Minimal Report size. Note that the Condition parameter, which tells if the analysis is valid, is set to Concur. Finally, the Variable centre range is set to 1-30, thus X will assume in the analysis the position of the centres X1, X2,...,X30 in ABC.</p>
	<p>Here is part of the results. The results indicate that for the first 30 centres the three Euler lines concur for X=X1, X4, X5, X17, X18.</p> <p>The point P for X=X1 was not found since it is either already labelled or nothing was found about it (in our case we find that P=X1).</p> <p>The point P for X=X4 is the centre X125 of ABC.</p> <p>The point P for X=X5 is the centre X140 of ABC.</p> <p>The point P for X=X17 is the centre X14138 of ABC.</p> <p>The point P for X=X18 is the centre X14139 of ABC.</p>

6.6. Object analysis with variable parameter

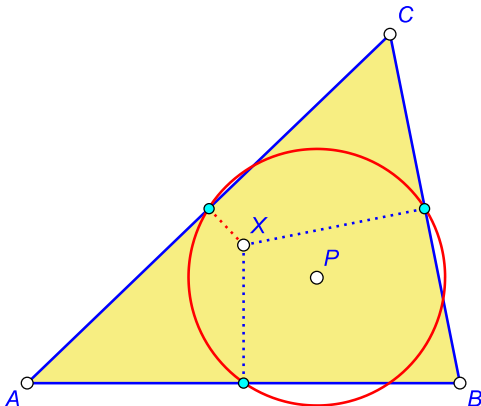
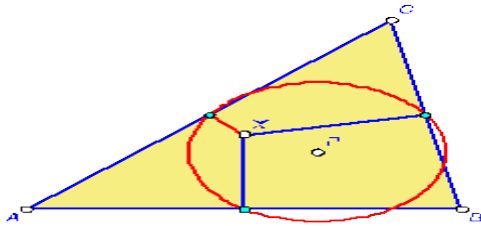
It is possible to perform a series of object analyses for constructions that depend on a parameter.

	<p>Let us observe the points with trilinear coordinates</p> $(b^n + (\sqrt{b}\sqrt{c})^n + c^n, c^n + (\sqrt{c}\sqrt{a})^n + a^n, a^n + (\sqrt{a}\sqrt{b})^n + b^n)$ <p>for $n=1,2,\dots,6$.</p> <p>In Sketch Editor we declare the parameter n (<i>Number/Value,Expression</i>) and a point P with trilinears as specified above (<i>Special/General triangle derived point</i>).</p> <p>Then switch to triangle analysis. Select Object</p>
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	<p>analysis as Type of analysis. Note how the Variable centre entry is declared. Set other options according to the desired details in analysis and report.</p> <p>Then press Study to run the analysis.</p>
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6.7. Fast variable centre analysis

Fast variable analysis is a quick way for checking if points constructed from triangle centres coincide with other known centres.

	<p>Given is a triangle ABC. For a point X, let P be the circumcentre of the pedal triangle of X in ABC. We examine if P is a known centre if X assumes the position of the first 1000 triangle centres.</p> <p>In the construction we define the point X to be the variable triangle centre X0 (available with the command <i>Special Triangle centre ETC X0</i>).</p> <p>We construct the circle through the projections of X onto the sidelines of ABC (it is also available as the pedal circle of a point in a triangle).</p> <p>Switch to the Triangle analysis module.</p>
	<p>Select Variable centre analysis as Type of analysis.</p> <p>In our analysis only the first 3500 centres are considered. If also the transformations of these centres (e.g. isogonal conjugation, isotomic conjugation) are to be considered, select the Extended treatment option.</p> <p>We choose All for Report size, otherwise only the first few results are displayed.</p> <p>The analysis shows if P matches any of the first 3500 centres if X is any of the first 1000 centres.</p>

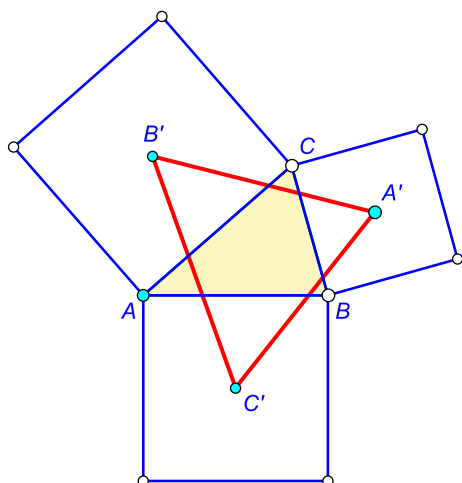
Variable centre analysis: P matches ABC centre: (14 items, reliable)

$P(X_1) = X_1$ (ABC)
 $P(X_2) = X_{597}$ (ABC)
 $P(X_3) = X_5$ (ABC)
 $P(X_4) = X_5$ (ABC)
 $P(X_6) = X_{597}$ (ABC)
 $P(X_{13}) = X_{396}$ (ABC)
 $P(X_{14}) = X_{395}$ (ABC)
 $P(X_{15}) = X_{396}$ (ABC)
 $P(X_{16}) = X_{395}$ (ABC)
 $P(X_{36}) = X_{1737}$ (ABC)
 $P(X_{40}) = X_{1158}$ (ABC)
 $P(X_{80}) = X_{1737}$ (ABC)
 $P(X_{84}) = X_{1158}$ (ABC)
 $P(X_{484}) = X_{1749}$ (ABC)

Exit Study Clear AddToNotes Show WhatIs AddToComment

Here is the list of matches. If we select Extended treatment option and/or if we increase the number of considered centres, then a much longer list appears. For example, we find that $P(X_{56}) = \text{Complement}(X_{2098})$ (ABC).

6.8. Two triangles analysis



Given is a triangle ABC. Consider the outward positioned squares based on the sides BC, CA, AB of the triangle ABC. The centres A' , B' , C' of these squares are the vertices of the outer Vecten triangle of ABC.

With two triangle analysis we examine if any centre of ABC matches any centre of $A'B'C'$.

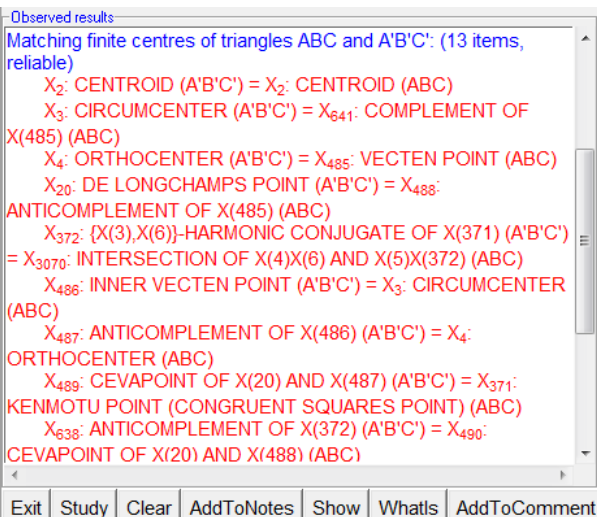
Type of analysis		Options	
<input type="radio"/> Object analysis <input type="radio"/> Variable centre analysis <input checked="" type="radio"/> Two triangle analysis		Extended treatment <input type="checkbox"/> Perspectivity centres <input type="checkbox"/> Short centre names <input type="checkbox"/> Report size <input type="text" value="All"/>	
Data for analysis			
Considered object(s)	A'B'C'		
Reference triangle	ABC		
Additional ref. points			
Consider centres X1 - X	3500		
Variable centre or parameter range			

Select **Two triangle analysis** as Type of analysis.

We choose All for **Report size**, otherwise only the first few results are displayed.

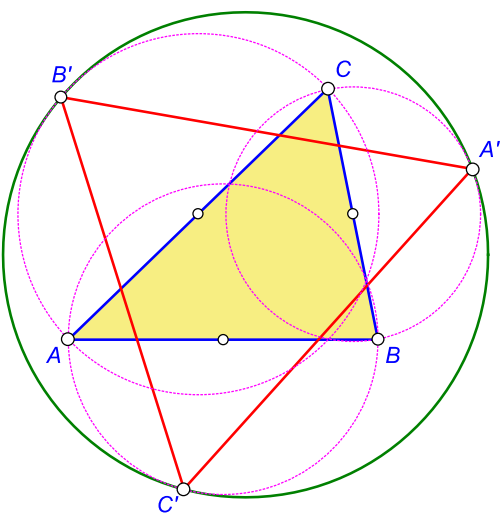
In our analysis only the first 3500 centres are considered. If also the transformations of the centres of $A'B'C'$ (e.g. isogonal conjugation, isotomic conjugation) are to be considered, select the **Extended treatment** option.

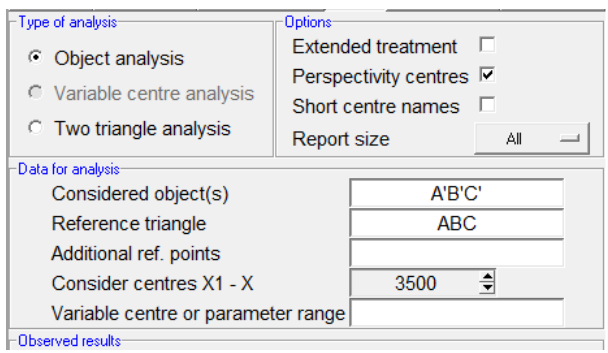
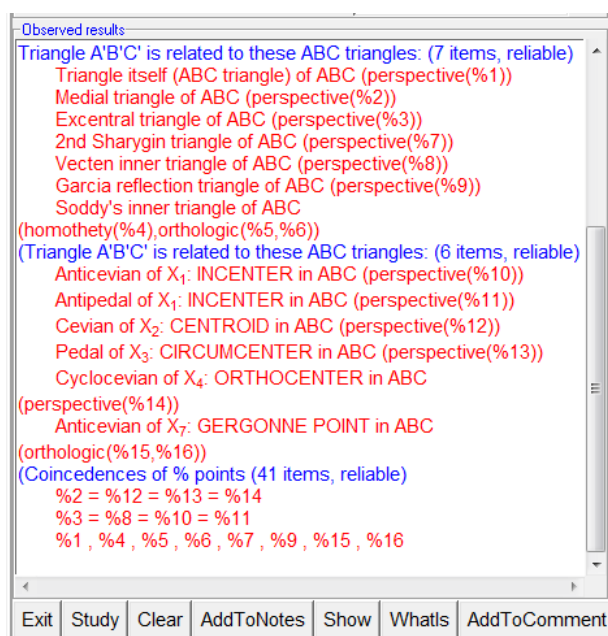
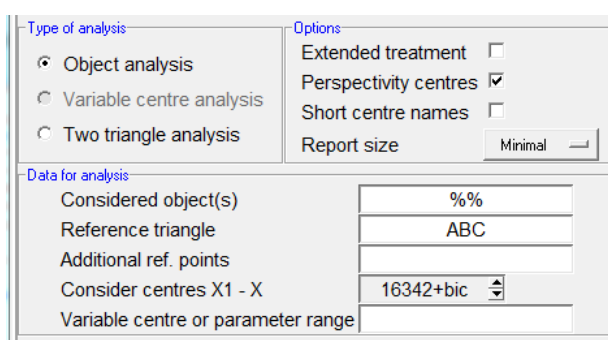
The analysis shows if any of the first 3500 centres of ABC matches any of the first 3500 centres of $A'B'C'$.

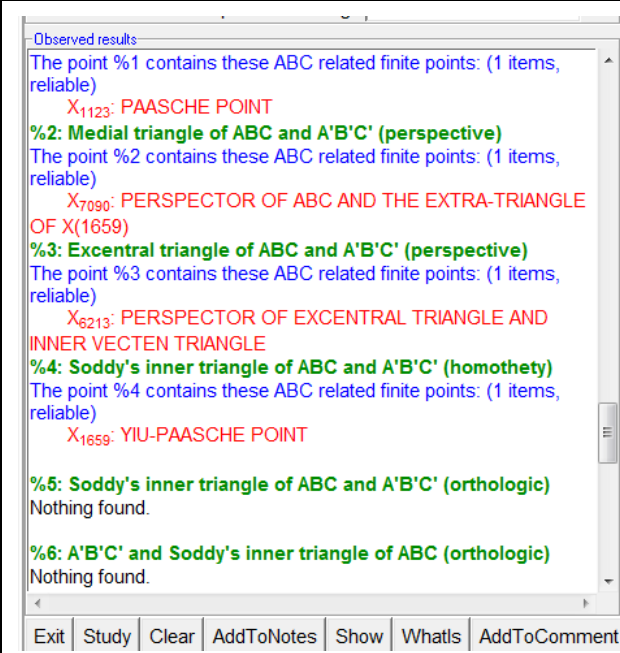
	<p>Here is the list of matches. If we select Extended treatment option and/or if we increase the number of considered centres, then a much longer list appears.</p>
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6.9. Triangle perspectivity analysis

With this analysis we investigate whether a triangle is perspective to different characteristic triangles of the reference triangle (e.g. the orthic triangle). The analysis considers about 200 characteristic triangles of the reference triangle. For each triangle it is checked, whether it is perspective, parallelologic, orthologic or cyclogic to the studied triangle. In case of a positive result, the perspectivity centres are found – they are denoted as %1, %2,.... The set of all found perspectivity centres is denoted with %.

	<p>Given is a triangle ABC. Consider the three circles having the sides BC, CA, AB as respective diameters. Let A', B', C' be the touch points of these circles with the circle that touches all of them internally.</p> <p>We examine if the triangle $A'B'C'$ is perspective to any of the characteristic triangles of ABC.</p>
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	<p>Select Object analysis as Type of analysis.</p> <p>We choose All for Report size, otherwise only the first few results are displayed.</p> <p>Select Perspectivity centres, so that the found perspectivity centres are stored as points %1, %2, ...</p> <p>Enter the reference triangle and the triangle to be studied (note that its circumcircles is studied as well).</p> <p>The Extended treatment and the Considered centres options are considered only subsequently when the %points are analysed.</p>
	<p>Here is the list of observed characteristic triangles of ABC that are homothetic, perspective, parallelologic, orthologic or cyclogic to A'B'C'. At the same time the respective centres are calculated.</p> <p>The calculated centres are listed at the end. Some of the centres coincide.</p>
	<p>We can study the centres %1, %2, ... as ordinary points.</p> <p>On the left we show how to study all of them at once (%% means the set of all found centres). Note that Report size was set to Minimal, otherwise the computation of detailed results takes time.</p> <p>This operation must be done before we Clear the results of the Perspectivity centres.</p>



Here is a section of the results. We see that X(1123) is the centre of perspectivity of ABC and A'B'C'. Use **What is** to learn about the Paasche point and **Show** to display it.

Note that nothing is found about %5. To obtain more observational data, enter %5 as Considered object, Increase Report size and use Extended treatment.

7. Examples

7.1. The Privalov conic

It is well known that in a given triangle ABC the vertices of the cevian triangles of any two different points D, E (not laying on the sidelines of ABC) are conconical (Figure 14 left). We shall consider the case of two special points: the Gergonne point (D) and the Nagel point (E) of a given triangle ABC. The Gergonne point of ABC is the point of concurrence of the line segments connecting the points where incircle of ABC touches the sides with the opposite vertices. Similarly, the Nagel point of ABC is the point of concurrence of line segments connecting the points where the excircles touch the sides of ABC with the opposite vertices.

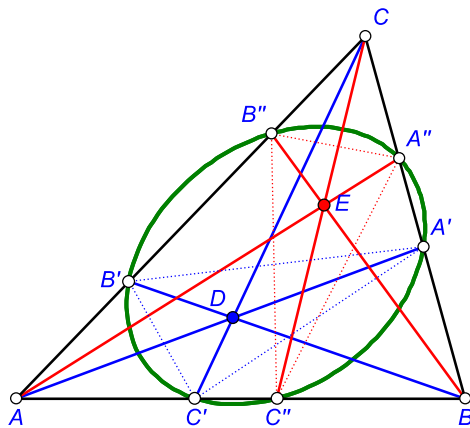


Figure 14

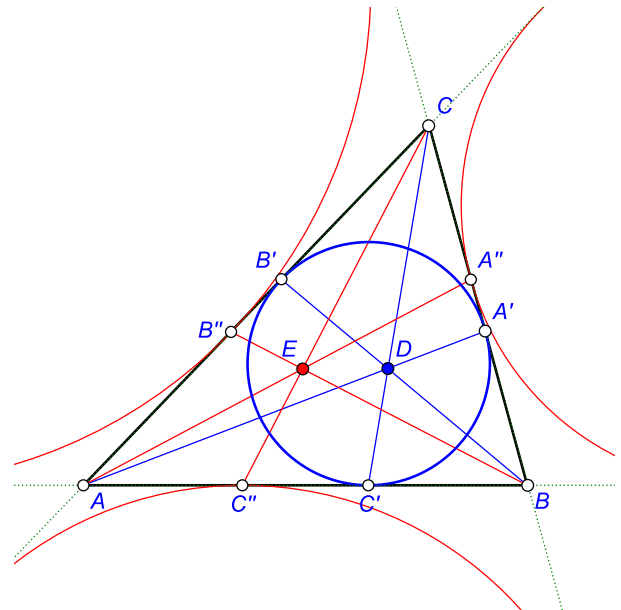


Figure 15

Here is a quick and efficient way of executing this construction:

Steps	Commands	Comment
1. Draw a triangle ABC	Point/Point (for A,B,C) Line/Polyline (to connect A-B-C-A) Special/Reference triangle	
2. Position the Gergonne point D and the Nagel point E of ABC	Special/Triangle centres/Advanced/Gergonne point Special/Triangle centres/Advanced/Nagel point	Keep the check-mark for using the reference triangle, since we construct the Gergonne and Nagel points of a previously defined reference triangle ABC.
3. Construct the Ceva-triangles of D and E	Special/Triangle point derived objects/Triangle/Cevian (choose then points D and E)	
4. Construct the ellipse and the characteristic points of the ellipse	Circle/Conic 5 pts Point/Conic points	Label the centre of the ellipse as S. (See Figure 16.)
5. Inspect the centre S of the ellipse	Special/Centre analysis (pick S)	OK Geometry observes that S is the ETC centre X5452 . (See Figure 17.) A more detailed inspection with

		<p>Triangle analysis suggest that, for example:</p> <p>S = Isogonal conjugate of the Cross conjugate of X6 and D, or</p> <p>S = Complement of the Isogonal conjugate of X1486, or</p> <p>S = Complement of the Isotomic conjugate of (X3434)</p>
6. Inspect the points on the constructed ellipse	Special/Centre analysis (pick the ellipse)	OK Geometry spots 3 centres along the ellipse (X3022, X3271, X4904). The extended search adds to this list 6 more transformed centres.

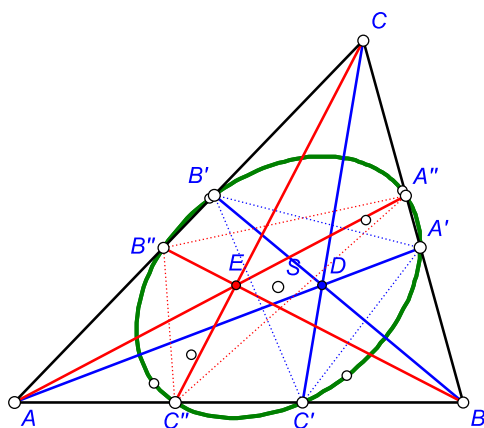


Figure 16

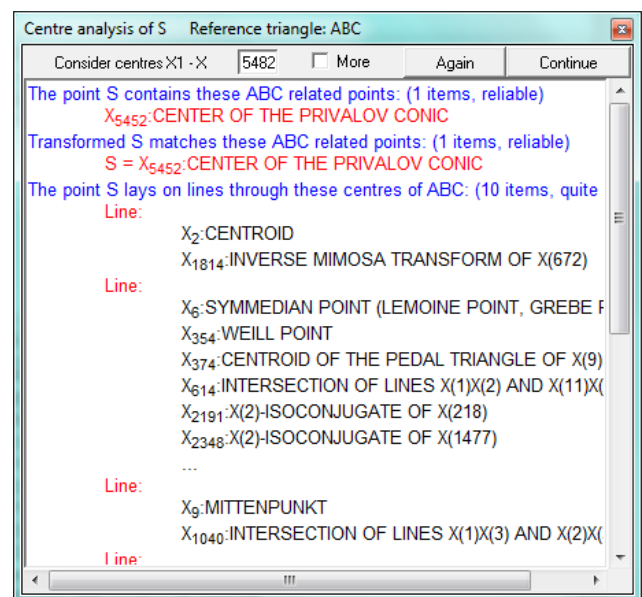


Figure 17

7.2. Circum-side-circle

Given is a triangle ABC. A circle having a side of ABC as its diameter is called a side-circle of ABC. Each triangle thus has three side-circles. We are looking for a way to construct the 'circumcircle' of the three side-circles of a given triangle, i.e. the three side-circles of ABC should touch from inside the constructed circle (Figure 18).

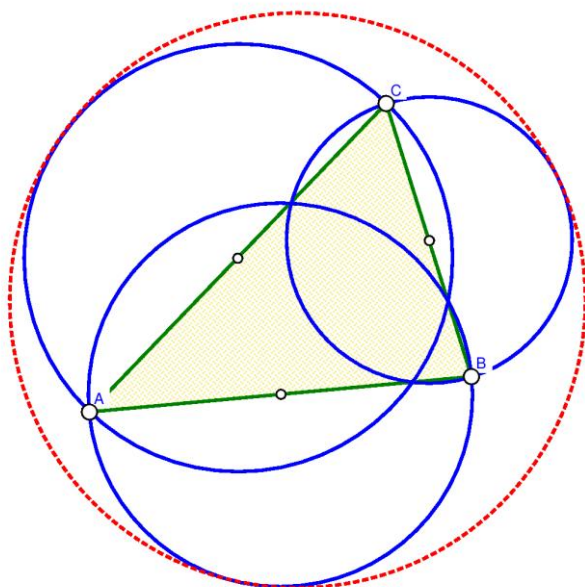


Figure 18

The strategy in the construction below is to find the three points of contact of the circum-by-circle with the three side-circles. Knowing how to construct (with Euclidean tools) these three points leads the way to the construction of the circum-by-circle.

Steps	Commands	Comment
1. Draw a triangle ABC	Point/Point (for A,B,C) Line/Polyline (to connect A-B-C-A) Special/Reference triangle	Try to make ABC visibly scalene.
2. Construct the three side-circles and the circum-side-circle	Point/Midpoint/(pick ABC)On each segment Circle/CircleC Circle/Circle 3 obj	Use repeatedly the Alt button (in the editor's menu bar to obtain the desired circle that is tangent to three objects.
3. Construct the three points of tangency A', B', C' of the circum-side-circle	Point/Point	The Observe command reveals that AA', BB', CC' are concurrent in a point, say, P.
4. Construct the lines AA', BB', CC' and the concurrency point P.	Line/Line 2 pts Point/Intersection	
5. Inspect the properties of the point P in	Set ABC as the reference triangle.	

reference triangle ABC	Enter the Triangle analysis module. Execute the Object analysis on the point P.	
6. Obtain information about the Paasche point	In the Triangle analysis module Position the cursor on X1123 and click the WhatIs button.	To obtain the basic information about the Paasche point position the cursor on this term and right click the WhatIs command (Figure 19).

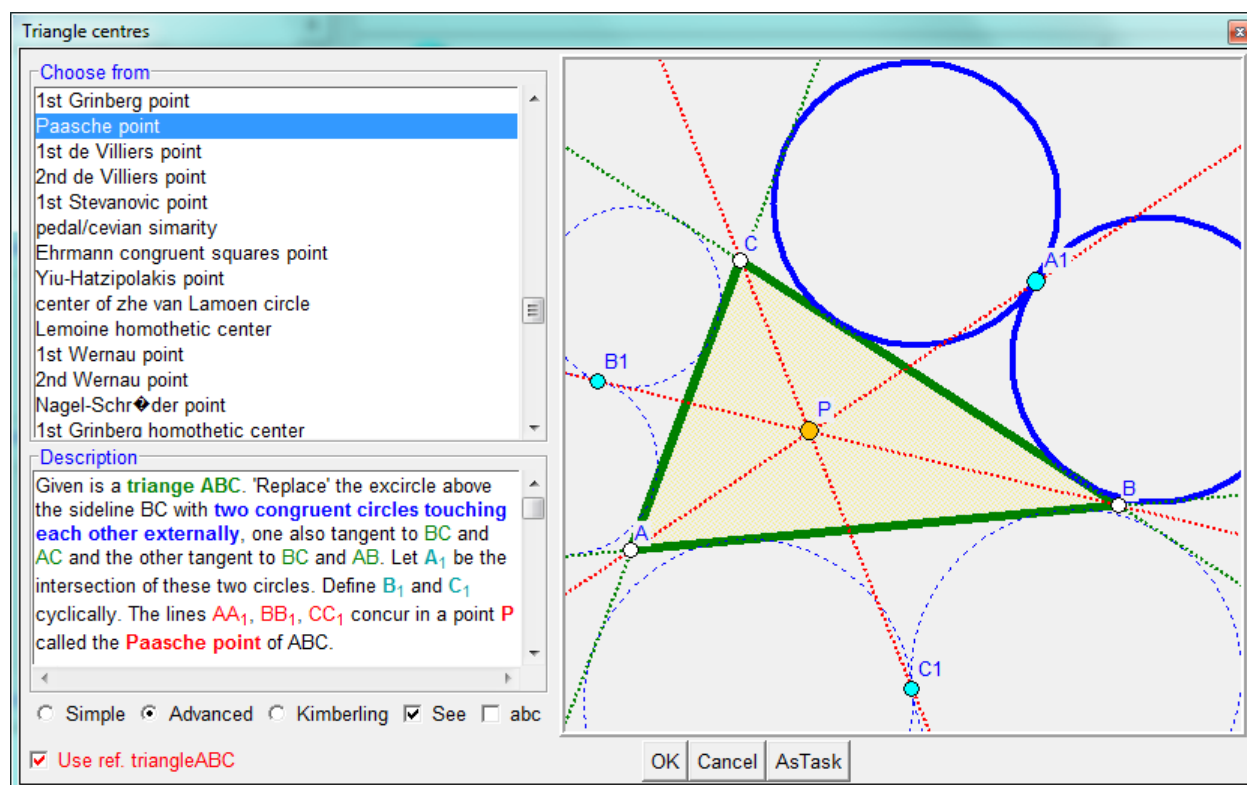


Figure 19

The Paasche point P of ABC can be constructed (using a homothety on a 'false' construction), and this gives way to the construction of the circum-side-circle. The construction steps are illustrated in Figure 20.

The correctness of the construction via the Paasche point can be verified with algebraic means.

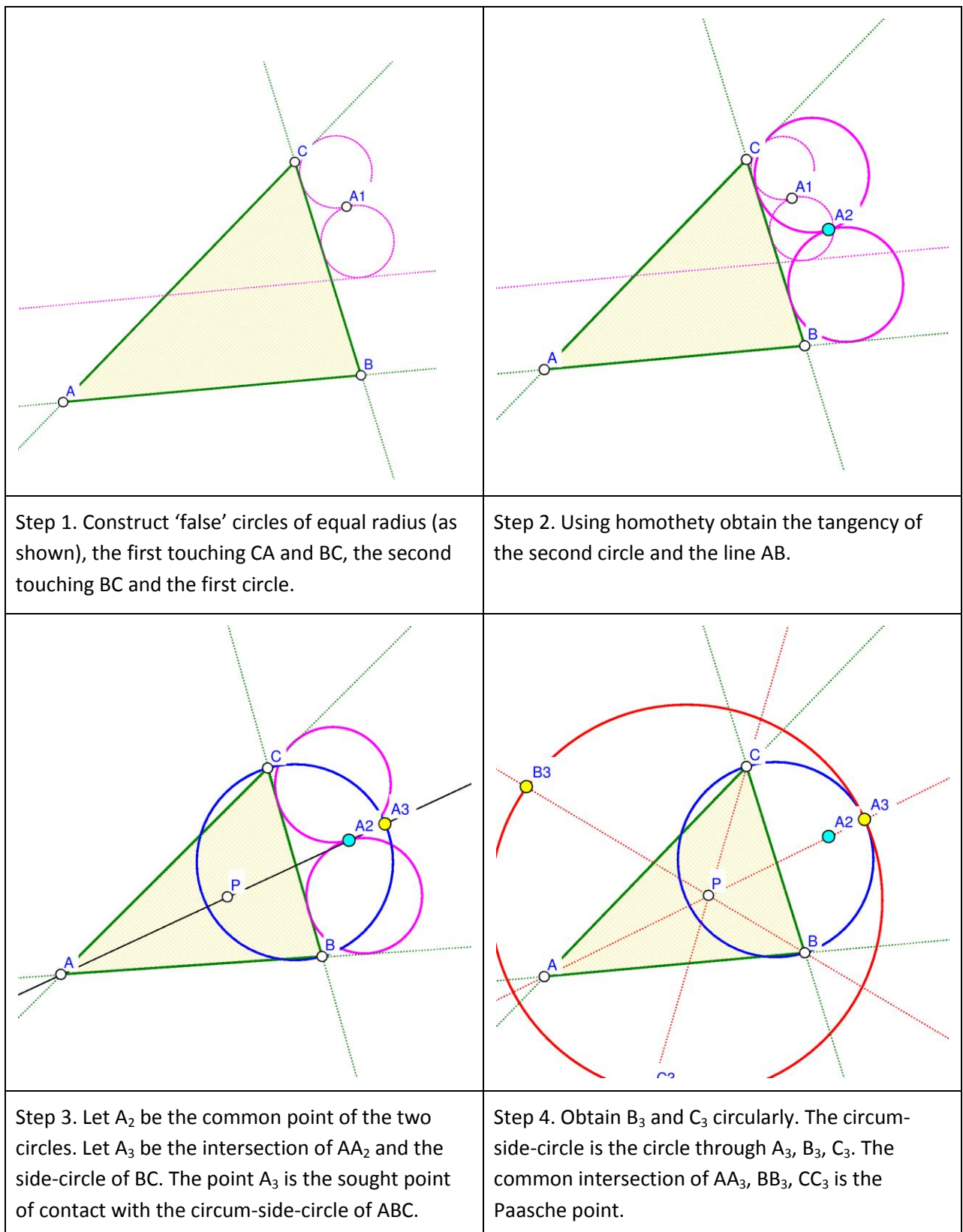


Figure 20

The circum-side-circle (Figure 18) is one of the 8 circles that are tangent to the three side-circles, see Figure 21. Here are some of the observed properties:

One of the 8 circles contains all three side-circles, this is the circum-side-circle with centre P_0 – the complement of the equal detour point $X(176)$.

Three of the 8 tangent circles are contained in exactly two of the side-circles, their centres are A_2, B_2, C_2 (see Figure 21). The lines AA_2, BB_2, CC_2 concur at point P_2 – the Yiu-Pasche point $X(1659)$.

Three of the 8 circles are contained in exactly one of the side-circles, their centres are A_1, B_1, C_1 (see Figure 21). The lines AA_1, BB_1, CC_1 concur at point P_1 – the isogonal conjugate of the 3rd Kenmotu homothetic centre $X(5414)$.

One of the 8 circles is contained all three side-circles. Its centre is P_3 – the complement of the isoperimetric point $X(175)$.

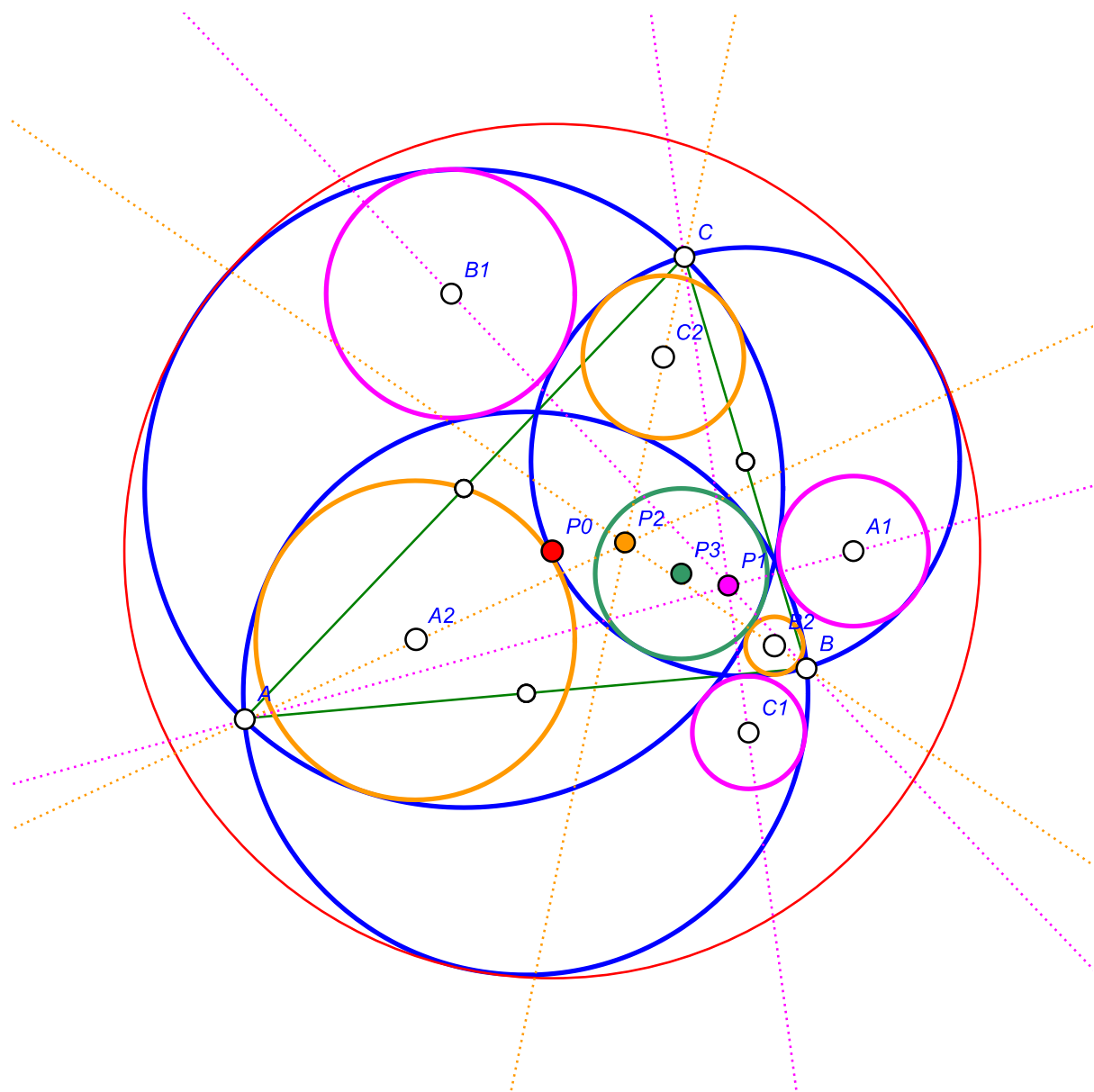
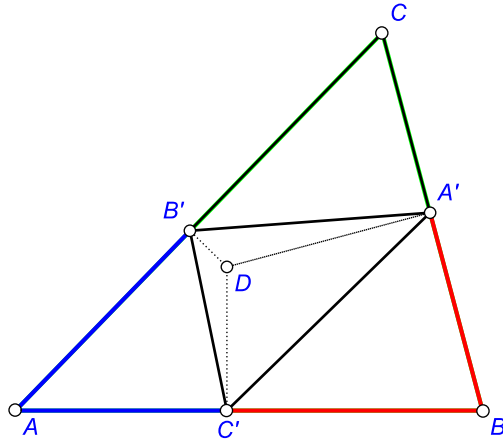


Figure 21

7.3. Trisecting triangle perimeter by pedal triangle



Given is a triangle ABC and a point D. The pedal triangle of D in ABC has as its vertices the projections A' , B' , C' of the point D onto the sidelines of ABC. We are looking for a point D with the property that A' , B' , and C' cut the perimeter of ABC into three pieces of equal length (Figure 22), so that $|B'A'| + |AC'| = |C'B'| + |BA'| = |A'C'| + |CB'|$.

The steps below lead to a reliable hypothesis for the sought position of the point D. The strategy is to (1) construct a general triangle, a point D and the related pedal triangle, (2) redefine D with an implicit construction, (3) analyse the new position of the point D, (4) make an improved check for the suggested position.

Figure 22

Steps	Commands	Comment
1. Draw a triangle ABC and a point D	Point/Point (for A,B,C) Line/Polyline (to connect A-B-C-A) Special/Reference triangle Point/Point (Construct the point D.)	Try to make ABC visibly scalene.
2. Construct the pedal triangle of D in ABC.	Special Triangle Point derived objects Triangle Pedal (Pick D.) Action Lables Label vertex (Label the vertices A' , B' , C' .)	Note that ABC was previously set as a reference triangle.
3. Construct the obtained 'pieces of perimeter' as polylines and measure their length.	Line Polyline (Pick B' , A , C' ; repeat the command for C' , B , A' and A' , C , B' .) Number Length circumference (Select the three constructed pieces.)	Name the lengths of the pieces as Len_A, Len_B, Len_C.
4. Check the sought condition	Advanced Check Equivalence SameValues3 (Pick the values Len_A, Len_B, Len_C.)	Name the check as SameValues3. Obviously, it turns out as False.
5. Implicit	Advanced Implicit construction	The implicit construction should

(re)construction of D.	(Uncheck the restriction (OK); then pick the condition SameValues3 (OK) and the point D.)	turn the condition to True.
6. Analyse the position of the point D	<p>Special Centre analysis (Pick D.)</p> <p>OK observes that D is the centre X(165) – the centroid of the excentral triangle.</p> <p>To obtain a description of terms used in the result, position the cursor on it, right-click and use the command WhatIs.</p>	Note that most items in the results are marked as ‘unreliable’, or even ‘great caution’. This happens when the construction is based on optimisation or implicit construction.
7. Check the construction directly	<p>Action Delete (pick the point D)</p> <p>Special Triangle centres ETC X165</p> <p>Repeat the steps 2-4.</p>	Note that the check is now much more accurate since there are no implicit points in the construction.

7.4. An analogy to Kosnita point construction

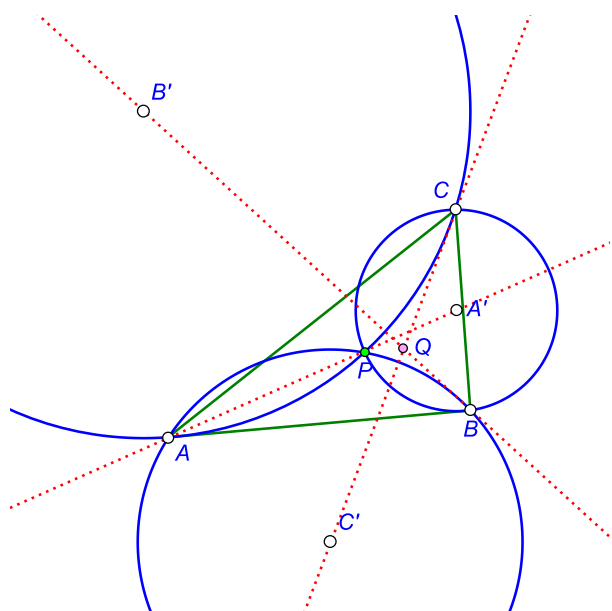


Figure 23

Given is a triangle ABC and a point P not laying on its sidelines. Let A' be the centre of the circumcircle of triangle PBC, and define B' and C' cyclically (Figure 23). In general, the lines AA' , BB' , and CC' do not concur. However, for certain points P this is the case. For example, when P is the incentre (X1) of ABC the mentioned lines concur in P. The lines concur also if P is the circumcentre of ABC – in this case the point of concurrence is the Kosnita point (X54) of ABC. Let us inspect for which triangle centres the lines AA' , BB' , CC' concur.

The strategy: (1) We shall construct a scalene triangle ABC. The point P will be set as a variable ETC centre. (2) Using the Variable centre analysis OK Geometry will produce hypotheses for which points the lines AA' , BB' , CC' concur, and what is the point of concurrence Q.

Steps	Commands	Comment
1. Draw a triangle ABC.	Point/Point (for A,B,C) Line/Polyline (to connect A-B-C-A) Special/Reference triangle (ABC)	Try to make ABC visibly scalene.
2. Construct the variable centre P. Set P to be X(2).	Special / Triangle centres / ETC / X0: VARIABLE CENTRE	Note the centre indicator that appears on the left pane. Set the indicator to 2 (so that P and Q will not coincide) and press Go .
3. Construct the three circles and their centres A',B',C'.	Circle / Circle 3obj (Pick P,B,C etc.) Point / Circle centre (Pick the three circles.) Label / Label vertex (Label the vertices as A', B', C')	
4. Construct the lines AA', BB', CC' and their eventual point of concurrence Q.	Line / Line 2 pts (Pick A,A', etc.) Point / Intersection (Let Q be the intersection of AA' and BB'). Advanced / Check property / Position / Concurrent lines (Pick the lines AA', BB', CC'. Name the condition of concurrency as ConcurrentL_1.)	Note that the Q is the point of concurrency only if ConcurrentL_1 is True.
5. Analyse the point Q for various centres.	In the Analysis module fill the entries as shown in Figure 24. After you press Study the observational results appear.	
6. B',C'.	Circle / Circle 3obj (Pick P,B,C etc.) Point / Circle centre (Pick the three circles.) Label / Label vertex (Label the vertices as A', B', C')	
7. Construct the lines AA', BB', CC' and their eventual point of concurrence Q.	Line / Line 2 pts (Pick A,A', etc.) Point / Intersection (Let Q be the intersection of AA' and BB'). Advanced / Check property / Position / Concurrent lines (Pick the lines AA', BB', CC'. Name the condition of	Note that the Q is the point of concurrency only if ConcurrentL_1 is True.

	concurrency as ConcurrentL_1.)	
8. Analyse the point Q for various centres.	In the Analysis module fill the entries as shown in Figure 24. After you press Study the observational results appear.	

Ok Geometry observes that the concurrency of lines occurs for 285 of the considered centres.

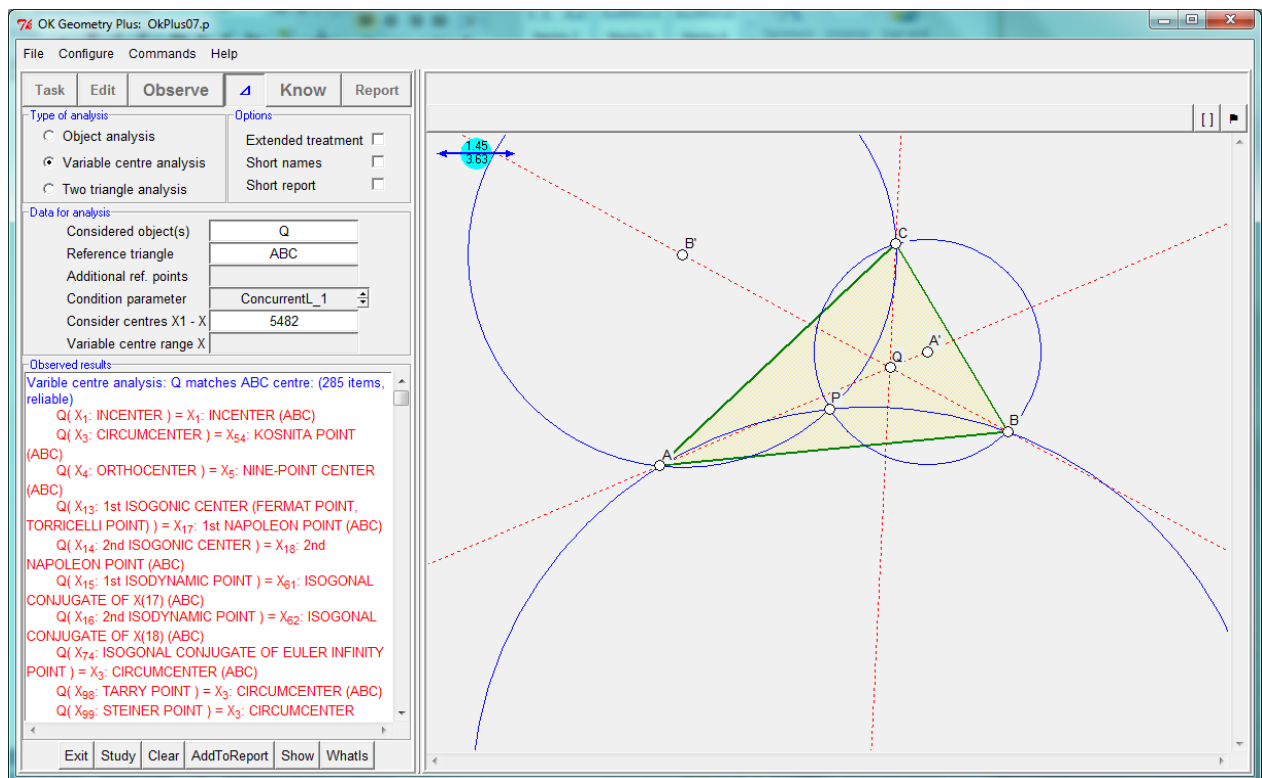


Figure 24